

LHC Project Note 242

December 8, 2000 Elena.Chapochnikova@cern.ch

Longitudinal beam parameters during acceleration in the LHC

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Keywords: Longitudinal beam stability, emittance blow-up

Summary

Longitudinal beam parameters in the LHC having been fixed at the lowest and highest energies, up to now still had some freedom during acceleration. In this paper they are defined during acceleration from beam stability considerations. The necessity of continuous emittance blow-up on the ramp proportional to the square root of energy is shown. The requirements for the damping of narrow-band impedances are specified. They are found to be most critical at injection energy.

1 Introduction

At the moment the longitudinal parameters of the LHC beam are fixed at the lowest and highest energies [1], [2]. On the flat bottom the longitudinal emittance is defined by the injector, SPS, and the capture system in the LHC as well as requirements from dynamic aperture. Parameters at the flat top come from optimisation of the luminosity and beam life time (intrabeam scattering). These boundary conditions leave at first view some freedom for the choice of parameters during the ramp. Below we show how the RF and beam parameters could be defined from longitudinal beam stability considerations - in particular, coupled-bunch instability thresholds.

2 Acceleration cycles

In this section we start from a general consideration of the possible different accelerating cycles in the LHC. The magnetic cycle is taken from [3]. It is composed of 4 differents parts - parabolic, exponential, linear and again parabolic growth of magnetic field. The last parabolic part, not being precisely defined in [3], is assumed here to start at the level of 97 %

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of the maximum field. For maximum magnetic field the last (updated) value is taken from the LHC web page [2] and is equal to 8.33 T.

The change in synchronous momentum through the cycle is proportional to the magnetic field variation:

$$p_s \left[eV/c \right] = ce\rho B \left[T \right], \tag{1}$$

where ρ =2803.93 m is a bending radius. The derivative of synchronous momentum determines the synchronous voltage (voltage seen by the synchronous particle) during the cycle by the equation:

$$eV_s = 2\pi R \frac{dp_s}{dt},\tag{2}$$

where $2\pi R$ is the machine circurference. The synchronous momentum and voltage are shown in Fig. 1.



Figure 1: The synchronous momentum (left) and the synchronous voltage (right) during the cycle.

For a given magnetic cycle, the voltage programme for a single harmonic RF system is fixed by the longitudinal beam emittance ε and filling factor q. The last parameter can be defined either as the ratio of the bunch emittance to the bucket area, q_a , or as the ratio of maximum momentum spread in the bunch to the bucket height, q_p . For a synchronous phase ϕ_s close to π (non-accelerating bucket) $q_p \simeq \sqrt{q_a}$.

For a given longitudinal emittance the filling factor $q \leq 1$ still can be considered as a free parameter. To find the optimum voltage programme (or define q) during the ramp, many different conditions should be taken into account. They include the maximum voltage and power available, beam stability, beam loading and others.

At injection into the LHC the emittance will be in the range from 0.5 to 1 eVs depending on how well intensity effects can be controlled in the SPS, since the nominal emittance at injection to the SPS is around 0.4 eVs, [4]. The SPS impedance reduction programme is underway to help control instabilities.

To have a clean bunch into bucket SPS-LHC transfer for emittances higher than 0.5 eVs, a 200 MHz RF system was proposed, to be used only for capture, in addition to the main 400 MHz RF system. It is based on four cavities, designed for a maximum voltage of 0.75 MV each, which can be pushed up to 1 MV, [5]. For capture the operational total voltage at 200 MHz is 3 MV [6]. In the present scenario after capture the voltage of the 400 MHz RF system is adiabatically increased up to 8 MV and the voltage of the 200 MHz RF system is decreased to zero.

On the flat top the emittance is required to be 2.5 eVs [1]. It is assumed that to satisfy this requirement the emittance will be blown up in a controlled way at some moment in the cycle. Another boundary condition is that the shortest possible bunch length is desirable during collision to increase luminosity. For this reason the maximum available voltage (16 MV) at 400 MHz will be applied at top energy producing ~1 ns long bunches.

First, let us consider the restrictions for different possible scenarios due to the maximum available voltage. In a subsequent section a consideration based on an analysis of the beam stability is also presented.

In Fig. 2 voltage programmes calculated both for 200 MHz and 400 MHz RF system used separately as a single RF system are presented for different emittances in the range from 0.5 to 2.5 eVs and fixed filling factor $q_p = 0.9$. Note, that these voltage programmes do not satisfy the "boundary conditions" at the flat bottom and flat top.



Figure 2: Voltage programme with fixed filling factor $q_p = 0.9$ for single 200 MHz (left) and 400 MHz (right) RF systems for different (constant) values of longitudinal emittance. Dashed line indicates hardware limit for voltage amplitude.

One can see that the 200 MHz RF system with 3 MV available can be used for acceleration of bunches with emittance ≤ 1.7 eVs from the beginning of the cycle and with an emittance

of 2.5 eVs from an energy higher than 1 TeV. However the minimum bunch length which can be obtained with 3 MV for this emittance on the flat top is 1.9 ns. In the following we will consider acceleration only with 400 MHz RF system.

Using the 400 MHz RF system alone (with 16 MV) is possible at the beginning of acceleration for emittances ≤ 1.4 eVs. The required voltage rapidly goes down with energy, so that bunches with emittances of 2.5 eVs can be accelerated from an energy of ~1.5 TeV.

The dynamic aperture of the machine could give another limitation to the time of emittance blow-up. In Fig.3 (left) the relative energy spread $\Delta E/E$ is shown for the voltage programmes presented in Fig.2 for the 400 MHz RF system. This energy spread corresponds in fact to the minimum value which can be obtained during the cycle for a given emittance keeping the voltage as low as possible (filling factor $q_p = 0.9$). The corresponding bunch length τ during the cycle which is maximum for a given emittance, is shown in Fig.3 (right). We will come back to this limitation later.



Figure 3: Relative energy spread (left) and bunch length (right) during the acceleration cycle for different values of longitudinal emittance and voltage programmes for 400 MHz RF system from Fig. 2.

In the next section we will define the optimum voltage programme and scenario for controlled emittance blow-up based on calculations of coupled bunch instabilities thresholds during the cycle and also at injection. This will allow us also to obtain requirements for HOM damping in the two RF systems of the LHC and any other narrow-band resonant impedances in the ring.

3 Beam stability

3.1 Narrow-band impedance

For equally spaced bunches the threshold for coupled-bunch instability due to a narrow-band resonant impedance with frequency $f_r = (pM + n)f_0 + mf_s$ [7], [8], can be approximately presented in the form:

$$R_{sh} < \frac{|\eta|E}{eI_0\beta^2} (\frac{\Delta E}{E})^2 \frac{\Delta\omega_s}{\omega_s} \frac{F}{f_0\tau} G(f_r\tau), \tag{3}$$

where f_0 is the revolution frequency, $f_s = \omega_s/(2\pi)$ is the synchrotron frequency, p = 0, 1..., n = 1, 2...M - 1, m = 1, 2... are integers, M is the number of bunches in the ring, $\eta = 1/\gamma_t^2 - 1/\gamma^2$, E is the synchronous energy, $\Delta E/E$ is the relative energy spread in the bunch, $\Delta \omega_s/\omega_s$ is the relative synchrotron frequency spread, $I_0 = MN_b e f_0$ is the average beam current, N_b is the bunch intensity and the form-factor $F \sim 0.3$ is defined by the particle distribution.

Function $G(x) = x \min\{J_m^{-2}(\pi x)\}$, where $x = f_r \tau$ and $J_m(x)$ is the Bessel function of order m, is shown in Fig. 4. At a given moment in the cycle the threshold is minimum $(G \simeq 1.5)$ for an impedance with frequency $f_r^{min} \simeq 0.43/\tau$.



Figure 4: Function $G(f_r\tau)$ from equation (3).

The instability thresholds calculated during the cycle using formula (3) are shown in Fig. 5 for emittances of 0.5 eVs, 1 eVs and 2.5 eVs and a voltage of 8 MV (left) and 16 MV (right). They are found for the resonant impedance frequency $f_r = f_r^{min}$ which corresponds to the worst case. Voltages of 8 MV and 16 MV are the nominal voltages in the 400 MHz RF system for the lowest and highest energies correspondingly. Emittance at injection can be in the range (0.5 - 1.0) eVs and on the flat top is fixed at the moment to the value of 2.5 eVs.

As one can see from Fig. 5, for a fixed emittance and voltage, the threshold shunt impedance decreases towards the end of the cycle. Indeed, for the worst-case frequency



Figure 5: Narrow-band impedance threshold R_{sh} for $f_r = f_r^{min}$ (solid line) together with threshold for the imaginary part of the broad-band impedance (dashed line) during the cycle for emittances 0.5 eVs, 1 eVs and 2.5 eVs and constant voltages of 8 MV (left) and 16 MV (right) in 400 MHz RF system. Thresholds correspond to nominal beam and bunch current, $I_0=0.56$ A and $I_b=0.2$ mA.

 f_r^{min} , it scales as ¹

$$R_{sh}^{thr} \propto \frac{\varepsilon^2 h^2}{E\tau} \propto \frac{\varepsilon^{3/2} V^{1/4} h^{9/4}}{E^{3/4}},\tag{4}$$

where h is the harmonic number (in the LHC for the 400 MHz RF system h = 35640).

Since at injection the emittance will be less than 1.1 eVs it should be increased in a controlled way to have 2.5 eVs at the flat top. This can be done neither on the flat bottom nor too early during the cycle due to the voltage being limited to 16 MV (see Fig.2).

It follows from Eq.(4) that to avoid decreasing the threshold during the cycle the emittance should be increased with energy not slower than

$$\varepsilon \propto E^{1/2} / V^{1/6}.$$
(5)

Note, that the bucket area also grows with energy as $E^{1/2}$. For 8 MV voltage at 450 GeV and 16 MV at 7 TeV in the 400 MHz RF system, we find from this scaling law that the emittance at the beginning of the ramp should not be less than 0.7 eVs. Here we assume that at the maximum energy, $E_{max}=7$ TeV, the emittance $\varepsilon_{max}=2.5$ eVs.

In Fig.6 (left) the threshold impedance corresponding to an emittance change

$$\varepsilon(E) = \varepsilon_{max} (E/E_{max})^{1/2} \tag{6}$$

is shown for a constant voltage of 8 MV and 16 MV. Then for $\varepsilon_{max} = 2.5$ eVs the initial emittance $\varepsilon(0.45 \text{ TeV}) = 0.63$ eVs.

¹For $\gamma_t = 53.7$ the slip factor changes very little during the cycle: from 3.43×10^{-4} to 3.47×10^{-4} .



Figure 6: Narrow-band (filled symbols) and broad-band (empty symbols) impedance threshold during the cycle for constant voltages of 8 MV (squares) and 16 MV (circles) - left figure, and variable voltage (shown as well with dashed line) - right figure. Emittance change according to (6) is shown with solid line. Thresholds correspond to nominal beam and bunch current, $I_0=0.56$ A and $I_b=0.2$ mA.

A linear change of voltage in time between 8 MV and 16 MV gives a threshold at 450 GeV slightly below the one at 7 TeV. This can be easily compensated by emittance blow-up taking into account the dependence on voltage in (5). For example, for voltage changing between 8 and 16 MV according to the formula

$$V = 16 \left(E/E_{max} \right)^{1/4} \, [\text{MV}],\tag{7}$$

we obtain from (5) the required emittance blow-up law

$$\varepsilon = \varepsilon_{max} (E/E_{max})^{11/24}, \tag{8}$$

which in practice is not very different from (6). Beam parameters during this "optimum" cycle are presented in Section 4.

To minimize particle losses, emittance blow-up should be performed for a bunch with weak nonlinearity and having enough free space in the bucket. Controlled emittance blow-up can be realised using band-limited noise introduced through the phase or amplitude loop as was done during $p\bar{p}$ operation in the SPS [9] and more recently in the KEK PS [10]. Details of possible scenarios will be discussed elsewhere.

3.2 Broad-band impedance

The stability criterion for narrow-band impedances analysed in the previous section, is derived ignoring the presence of any other impedance in the ring and assuming that there is Landau damping. This condition is not satisfied if the coherent frequency shift of the given azimuthal mode m due to the broad-band impedance is larger than one fourth of the synchrotron frequency spread [7]. Accurate analysis of stability criteria derived for reactive impedances with a small resistance treated as a perturbation, [11], gives finally a similar condition which can be written in the form

$$|\mathrm{Im}Z|/n < \frac{|\eta|E}{eI_b\beta^2} (\frac{\Delta E}{E})^2 \frac{\Delta\omega_s}{\omega_s} f_0 \tau,$$
(9)

where $I_b = N_b e f_0$ is the bunch current.

The preservation of natural Landau damping is especially important in the absence of a longitudinal feedback system [12].

During the cycle the threshold changes as

$$\text{Im}Z^{thr}/n \propto \frac{\varepsilon^2 \tau h^2}{E} \propto \frac{\varepsilon^{5/2} h^{7/4}}{E^{5/4} V^{1/4}}.$$
 (10)

For a constant emittance the threshold quickly drops down with energy, as can be seen in Fig. 5.

The necessity for emittance blow-up on the ramp, to avoid loss of Landau damping due to the broad-band impedance, is also discussed in [13].

Neglecting the weak dependence on voltage amplitude, we find from (10) that the condition to avoid decrease of the threshold (10) during the cycle,

$$\varepsilon \propto E^{1/2} V^{1/10},\tag{11}$$

is similar (but slightly less strong) to the one found above for the narrow-band impedance, see (5).

The threshold for the imaginary part of the broad-band impedance with a change of emittance during the cycle $\propto \sqrt{E}$ is shown in Fig. 6 for constant voltages of 8 MV and 16 MV (left) as well as for a voltage changing linearly between these 2 values (right).

Even for ultimate bunch intensity ($I_b = 0.3 \text{ mA}$) the minimum threshold value during the cycle 0.5Ω is well above the present estimation of the inductive part of the LHC broad-band impedance, [14], [12], which is

$$\mathrm{Im}Z/n \simeq 0.15 \ \Omega.$$

Additional margins may come from the fact that in this consideration the effect of incoherent frequency spread was not taken into account.

However as we will see below the most critical area is, in fact, beam stability at injection.

3.3 Beam stability on the flat bottom

The present scenario for injection into the LHC is the following [6]:

- capture in the 200 MHz RF system with 3 MV voltage; during this period the 400 MHz RF system is used for wave-form linearization with 0.75 MV;
- adiabatic bunch transfer from 200 MHz to 400 MHz by increasing the voltage in the 400 MHz RF system up to 8 MV and decreasing the voltage in the 200 MHz system to zero;

• start of acceleration with the 400 MHz alone, the 200 MHz being actively or passively damped.

The emittance of bunches coming from the SPS can be in the range (0.5 - 1.0) eVs. For larger emittances it will be difficult to avoid losses during transfer to the 400 MHz RF system.

Let us consider first the beam stability in the 200 MHz RF system alone. Assuming matched conditions, from formulae (3) and (9) we obtain the limitations shown in Fig.7 as a function of emittance.



Figure 7: Narrow-band and broad-band impedance threshold at 450 GeV as a function of emittance in 200 MHz, 3 MV (left) and 400 MHz, 8 MV (right) RF systems. Thresholds correspond to nominal beam and bunch current, $I_0=0.56$ A and $I_b=0.2$ mA.

For the same emittance the thresholds in the 200 MHz RF system are lower than in the 400 MHz due to decreased nonlinearity and therefore synchrotron frequency spread (proportional to h^2), see (4) and (10). However one should take into account that the lower harmonic, 200 MHz RF system, allows capture without losses of emittances at least twice higher than the 400 MHz RF system (bucket area is proportional to $h^{-3/2}$). Then the stability conditions in the two RF systems are comparable. Capture in the 400 MHz RF system alone can be done only for emittances around or below 0.6 eVs and even then only with the installation in the SPS of an additional, 400 MHz RF system [6].

The limitation for broad-band impedance at injection is the same as at 7 TeV with 2.5 eVs emittance only for emittances around 0.9 eVs. For low emittances (0.5 - 0.6 eVs) the risk of losing Landau damping due to the broad-band impedance becomes significant.

To improve the situation some emittance blow-up can be performed, if necessary, either in the SPS or in the LHC. In the last case this can be due to filamentation of an unmatched bunch or be done artificially after capture, for each batch separately.



Figure 8: Voltage, synchronous phase in degrees at 400 MHz (left), and emittance (right) during the cycle.

As was mentioned above for emittances around 1 eVs it is foreseen to use the 400 MHz RF system for wave-form linearization which should decrease filamentation and provide a better capture into the 200 MHz RF system. In this case one can expect a decrease in the thresholds of around 10% in comparison with the limitations presented in Fig. 7 (left). For lower emittances wave-form linearization is less necessary and therefore the 400 MHz RF system can be used to increase Landau damping - in bunch lengthening or bunch shortening mode.

Note that in the case when the impedances are close to their thresholds an accurate analysis should include the two impedances (broad-band and narrow-band) simultaneously.

In the case of loss of Landau damping the worst growth rates we can expect on the flat bottom (bunches in the 200 MHz RF system) for shunt impedances of 100 k Ω and nominal beam current are of the order of $\omega_s \times 10^{-2}$ s⁻¹. This corresponds to a typical growth time close to one second.

4 Beam parameters during the "optimum cycle"

In Fig. 8 we present voltage and emittance change programmes suggested as an example in section 3.1, see (7) and (8). This acceleration cycle provides

- the required bunch parameters at flat bottom and flat top;
- constant stability conditions during acceleration for the narrow-band impedance and no degradation for the broad-band impedance, with minimum controlled emittance blow-up;

• a small filling factor during the ramp to ensure emittance blow-up without losses.

If at the beginning of the ramp the emittance is more than 0.7 eVs, the beam should be blown up only from the point where the emittance becomes less than the emittance from the blow-up curve in Fig. 8 (right).

The variation of the synchrotron frequency spread during the acceleration cycle is shown in Fig. 9 (left) together with the energy spread. As one can see the relative synchrotron frequency spread is always below 0.3. This is important for blow-up using band-limited noise excitation at the quadrupole synchrotron frequency. The minimum width of the spectrum in frequency is only a few Hz. The synchrotron frequency f_s during the cycle is shown in Fig. 9 (right).



Figure 9: Relative energy and synchrotron frequency spread (left) together with synchrotron frequency f_s (right) corresponding to voltage programme and emittance change shown in Fig.8.

These voltage and emittance programmes, contrary to the linear variation of voltage shown in Fig. 6 (right), do not give any increase in energy spread at the beginning of ramp and should not be a problem for the LHC dynamic aperture [15].

The corresponding bunch length and filling factor are shown in Fig.10 (left). For this voltage programme the bunch does not occupy more than 60% of the bucket area. For an initial emittance of 1 eVs at the beginning of the ramp the filling factor $q_a = q_p^2 = 0.8$, see Fig. 2.

These voltage and emittance programmes provide threshold impedances for R_{sh}^{min} (minimum value at $f_r = f_r^{min}$) and ImZ/n as shown in Fig. 10. As expected there is no degradation of thresholds during the cycle compared to the fixed value on the flat top.

Even with the threshold impedances R_{sh}^{min} and ImZ/n at injection being below those on the flat top (for 2.5 eVs bunches), emittance blow-up might be still necessary during the



Figure 10: Bunch length and RF bucket filling factor (left) corresponding to voltage and emittance programmes shown in Fig. 8 together with narrow-band and broad-band impedance threshold (right).

cycle. Indeed, for narrow-band impedances for resonant frequencies above 400 MHz, R_{sh}^{thr} is decreasing as the bunch shrinks from (2 - 2.5) ns at injection to ≤ 1 ns on the flat top, see Fig. 4. Values of R_{sh}^{thr} as a function of resonant frequency for 0.5 eVs and 1 eVs emittances in 200 MHz RF system at injection are shown in Fig. 11 together with the limitation on the flat top for 2.5 eVs bunches in 400 MHz RF system. As one can see, for high resonant frequencies the limitation on the flat top for 2.5 eVs emittances around or more than 1 eVs due to bunch length variation.

Without emittance blow-up the limitation for ImZ/n during the cycle, see Fig. 5, also quickly drops down below its injection level in Fig. 7.

The thresholds for an emittance of 0.7 eVs in 200 MHz (with voltage of 3 MV) and in 400 MHz (with voltage of 8 MV) RF systems are shown in Fig. 12 for comparison together with the threshold on the flat top for an emittance of 2.5 eVs in 400 MHz RF system.

Note that criterion (3) is derived for equally spaced bunches. However it was applied above, using the nominal average beam current, for an LHC beam which in fact consists of many batches and gaps. For low-limit estimation of thresholds one should therefore introduce a factor of $M * 10/h \simeq 0.8$. Another factor 0.5 for threshold can come from considering different types of particle distribution, see [8]. As was mentioned above using linearization of the wave-form at injection for large emittances also brings narrow-band impedance threshold down by factor 0.9.

This means that to be safe for the nominal intensity for emittances starting from 0.7 eVs and for ultimate intensity for an emittance of 1.0 eVs, one should limit the shunt impedance to 60 k Ω in the frequency range (100 - 400) MHz. The limitation for R_{sh} then increases with frequency as $f_r^{5/3}$.



Figure 11: Limitation on shunt impedance of narrow-band resonances as a function of their frequency for emittances of 0.5 eVs (lowest curve) and 1 eVs in 200 MHz RF system (with 3 MV) at 450 GeV together with the limitation for 2.5 eVs bunches in 400 MHz RF system (with 16 MV) at 7 TeV (dashed curve). Thresholds correspond to nominal beam current $I_0=0.56$ A.

In fact the limiting curve for 0.7 eVs emittance in 200 MHz, see Fig. 12, scaled down by the factor 0.4, can be considered then as the requirement for HOM damping.

The Keil-Schnell-Boussard criterion for microwave instability gives the limitation for the broad-band impedance |Z|/n which is $\omega_s/\Delta\omega_s$ times higher than the limitation for ImZ/n from formula (9) and during the cycle presented in this section has the constant value around 7 Ω . In the same way as for ImZ/n, the minimum limitation for |Z|/n is reached on the flat bottom and for an emittance of 0.5 eVs in 200 MHz RF system is 1.3 Ω for the nominal bunch intensity.

5 Conclusions

For the LHC acceleration cycle, voltage and longitudinal emittance blow-up programmes are suggested which satisfy boundary conditions on the flat bottom and flat top and the provide required beam stability.

Without continuous emittance blow-up during the acceleration cycle the threshold of coupled-bunch instability decreases. At the same time the danger of losing Landau damping due to the broad-band impedance increases.

For an initial emittance of 0.7 eVs a controlled emittance blow-up proportional to \sqrt{E} provides a constant threshold during the ramp for the narrow-band impedances and no



Figure 12: Limitation on shunt impedance of narrow-band resonances as a function of their frequency for an emittance of 0.7 eVs at 450 GeV in 200 MHz RF system with 3 MV (lowest curve) and in 400 MHz RF system with 8 MV (upper curve) together with the limitation for 2.5 eVs bunches in 400 MHz RF system (with 16 MV) at 7 TeV (dashed curve). Thresholds correspond to nominal beam current $I_0=0.56$ A.

degradation of stability due to the broad-band impedance in comparison with the fixed value on the flat top.

With this emittance blow-up during the cycle the most critical area, which defines the requirements for HOM damping, is the beginning of the flat bottom.

At injection, emittance can be in the range (0.5 - 1) eVs. For clean bunch capture and transfer from the 200 MHz to 400 MHz RF system smaller emittances are desirable. However large emittances are preferable for beam stability. A compromise will be found during commissioning and operation.

For ultimate beam intensity and an emittance of 1 eVs the requirement to damp narrowband resonances to below 60 k Ω is obtained in the frequency range (100 - 400) MHz. For higher frequencies the limitation increases as shown in Fig. 12. For nominal beam intensity, with this limitation for shunt impedances, emittances on the flat bottom can be in the range (0.7 - 1) eVs.

For smaller emittances the beam stability at injection can be also improved by using the 400 MHz RF system as a Landau cavity (to increase synchrotron frequency spread).

Acknowledgments

Many thanks to T. Linnecar for useful discussions and important suggestions. The author is also grateful to D. Brandt, F. Ruggiero, J. Tuckmantel and L. Vos for valuable comments.

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