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Longitudinal and Transverse Modes in the CMS Experimental Chamber

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Summary

In a previous note, we calculated the loss factor and the longitudinal narrow band impedance associated with the CMS experimental chamber in the LHC and calculated the associated rise times for Robinson instabilities. The aim of this note is to extend the analysis to the transverse impedance and to estimate the resulting rise times for arbitrary multi-bunch instabilities and to compare the results of different computer codes.

1 Introduction

Fig. 1 shows the upper half of the original design for the CMS experimental chamber. This original design leads to a rather large loss factor and in [1] it was suggested to lower the loss factor by introducing an additional tapering at the beginning and end of the experimental chamber. Fig. 2 shows the modified experimental chamber with an additional tapering of 2 m at each side of the chamber and Fig. 3 shows the loss factor as a function of the tapering length x. It should be pointed out that the radial and longitudinal dimensions are not identical. The maximum radius of the experimental chamber is only 25 cm, whereas the total length of the chamber is 25 m. The large difference between the radial and longitudinal dimensions imposes some difficulties for the numerical calculations of the wake fields and the validity of the numerical calculations must be carefully analysed.

The aim of this note is to calculate the shunt resistance and Q-values for the longitudinal and transverse modes in the CMS experimental chamber with URMEL [2] and MAFIA [3], two computer codes for the computation of resonant modes, and to compare the results for the longitudinal modes with the results from the ABCI [4] and SUPERLANS [5] calculations in [1]. ABCI solves Maxwell's equations directly in the time domain for arbitrary charge distributions. Using a moving mesh, ABCI drastically reduces the number of mesh points and thus, allows the calculation of wake potentials in very long structures like the CMS experimental chamber. In [1] we calculated the loss factor with ABCI using different mesh sizes and found that the loss factor did not vary much with the mesh size. However, even though ABCI allows a precise calculation of the loss factor, it does not yield independent estimates for the shunt resistance

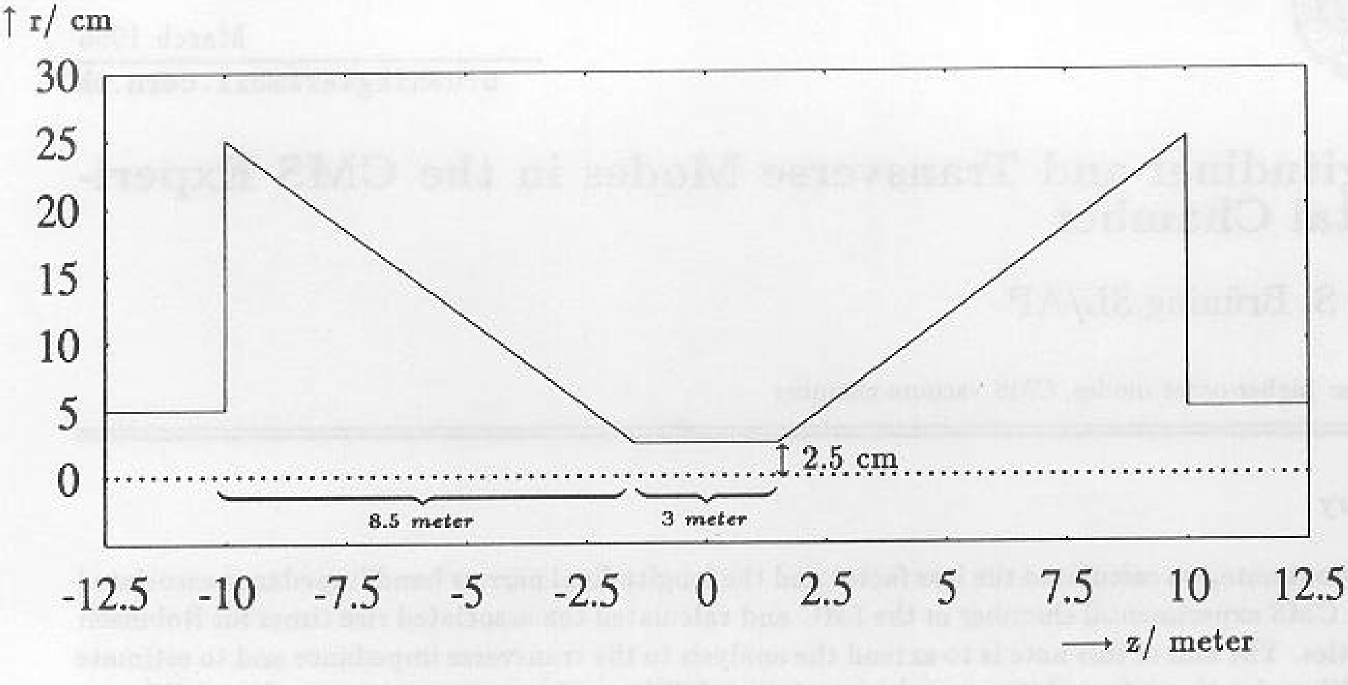

Figure 1: *The LHC vacuum chamber at the interaction points. The vacuum chamber has rotational symmetry and the picture shows only the upper half of the chamber. Note the differing radial and longitudinal dimensions.*

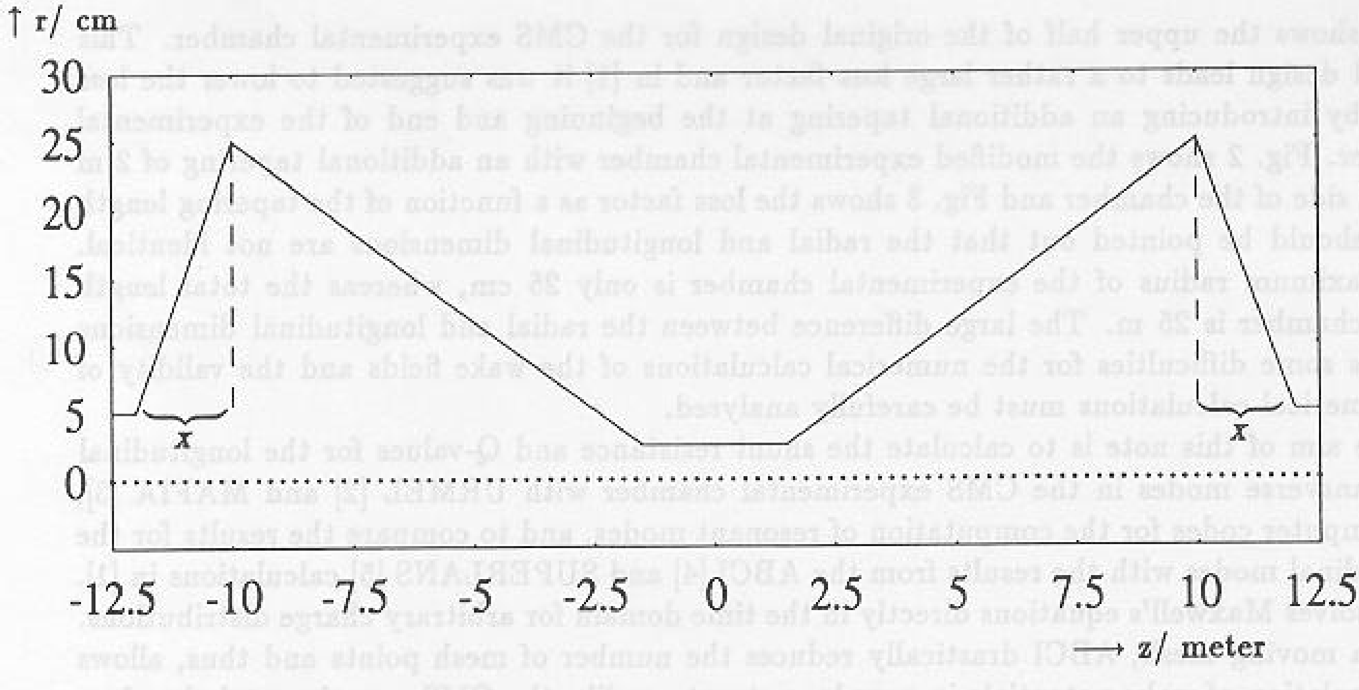

Figure 2: *The LHC vacuum chamber at the interaction points with a tapering of 2 m. The slope of the vacuum chamber at the beginning and the end of the experimental chamber is determined by the tapering length x. The vacuum chamber has rotational symmetry and the picture shows only the upper half of the chamber. Note the differing radial and longitudinal dimensions.*

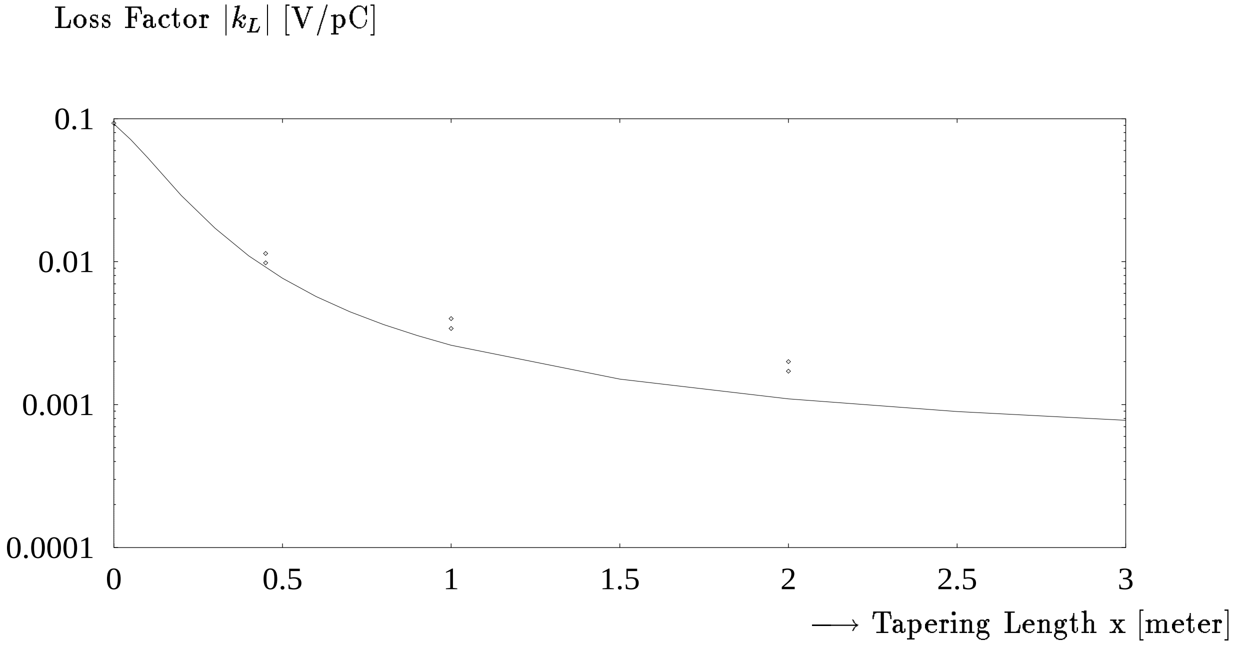

Figure 3: *The absolute value of the loss factor k_L versus the tapering length x of the experimental chamber in meter. The solid line is calculated using ABCI. The points are calculated with the MAFIA program. In both cases, the bunch length is 7.5 cm. Note the logarithmic scale on the vertical axis.*

and the Q-value of a resonance. Only the quotient of the shunt resistance and the Q-value can be estimated from the frequency spectrum of the loss factor in ABCI. Thus, one needs additional programs like MAFIA, URMEL, or SUPERLANS for an estimate of the shunt resistance and the Q-values. Because these programs might have difficulties calculating modes in a long structure, we must first check the validity of these codes. Taking the ABCI results for the loss factor as a reference, we first compare the calculated loss factor and its frequency spectrum of SUPERLANS, URMEL, and MAFIA with the results of the ABCI calculations. Once we have checked the validity of the trapped mode calculations, we will use the numerically calculated shunt resistance and Q-values for an estimate of the rise times for the transverse and longitudinal multi-bunch instabilities.

2 Comparison of Different Numerical Codes

In a first step, we will check the validity of the URMEL, MAFIA and SUPERLANS calculations by comparing the frequency spectrum of the longitudinal trapped modes and the resulting longitudinal loss factor with the results obtained with ABCI. ABCI calculates the loss factor from the wake potential

$$k_L = -\frac{1}{Q_b^2}\int_{-\infty}^{+\infty} dz^{'}\rho(z^{'})\int_{-\infty}^{+\infty} dz\rho(z)W_L(z^{'}-z), \tag{1}$$

where Q_b is the total charge per bunch, $\rho(z)$ the longitudinal charge distribution, and W_L the longitudinal wake function of a point charge. Fig. 4 shows the frequency spectrum of the ABCI

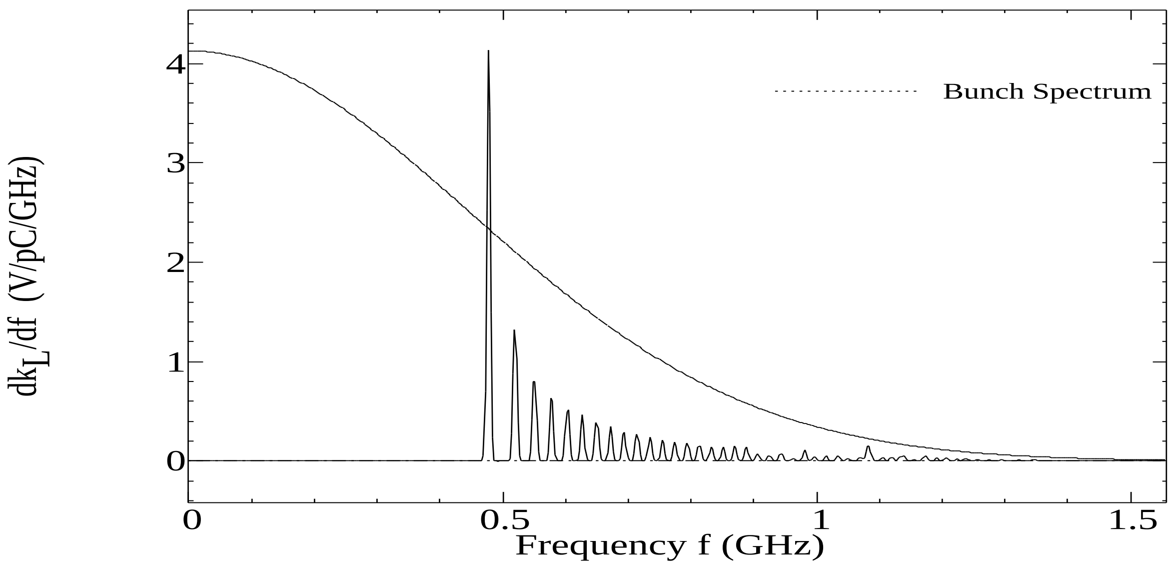

Figure 4: *The frequency spectrum of the ABCI loss factor for the CMS experimental chamber* **without tapering**. *The mesh size for the ABCI Calculations is 5 mm in both directions and the bunch length 7.5 cm.*

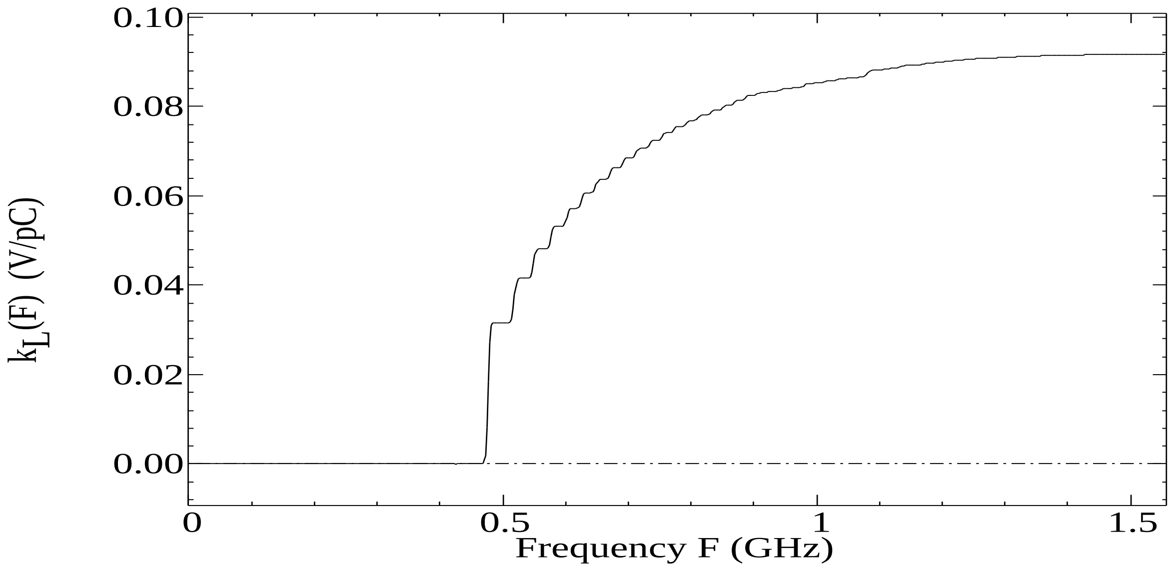

Figure 5: *The sum of the frequency contributions in the ABCI loss factor for the CMS experimental chamber* **without tapering** *as a function of the maximum frequency. The mesh size for the ABCI calculations is 5 mm in both directions and the bunch length 7.5 cm.*

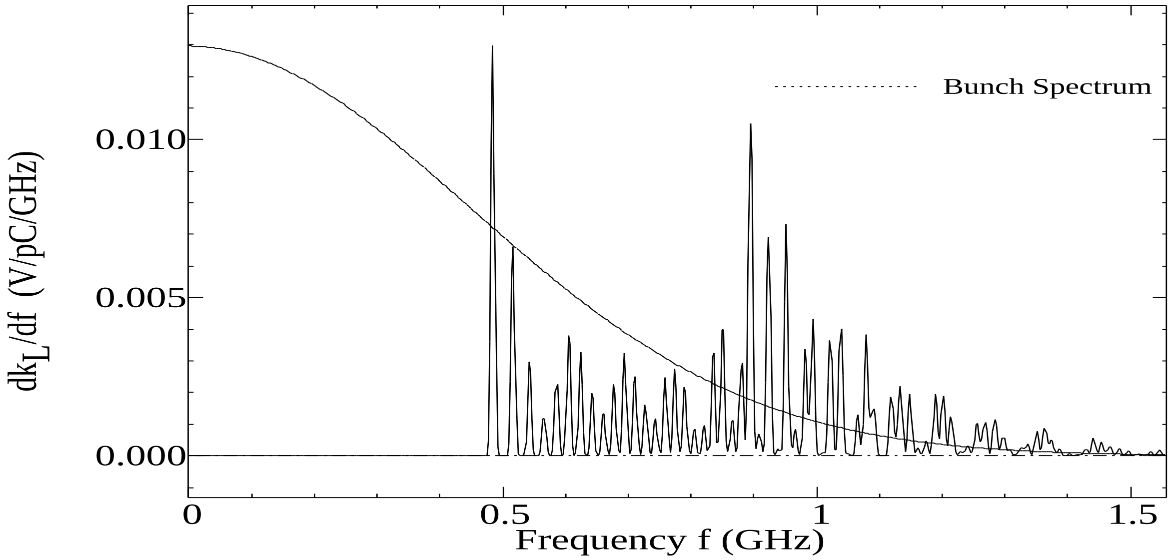

Figure 6: *The frequency spectrum of the ABCI loss factor for the CMS experimental chamber* **with tapering of 2 meters on each side of the structure**. *The mesh size for the ABCI Calculations is 5 mm in both directions and the bunch length 7.5 cm.*

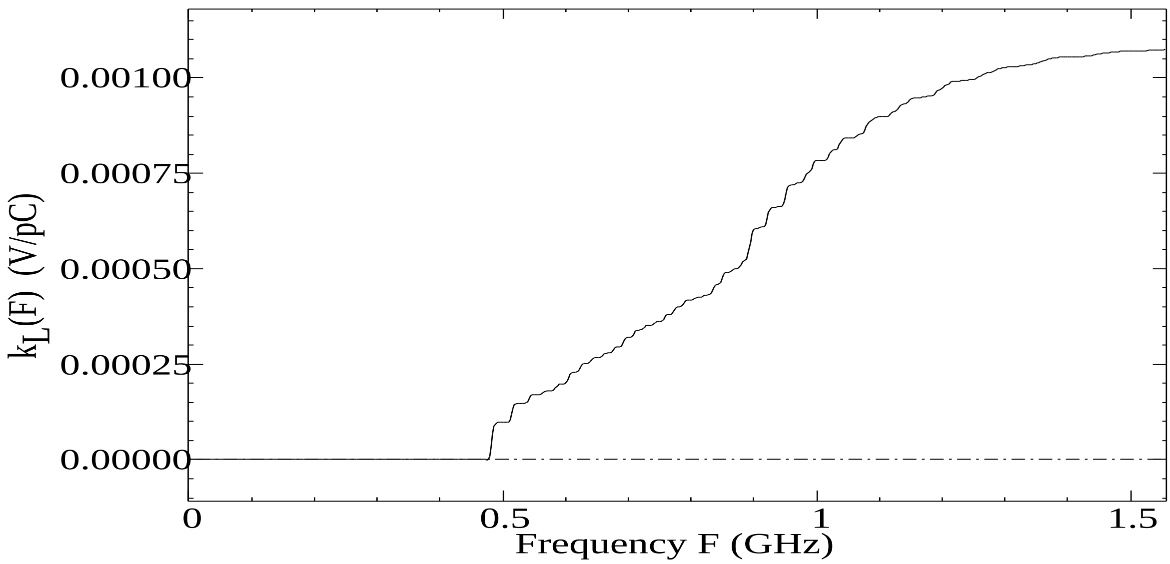

Figure 7: *The sum of the frequency contributions in the ABCI loss factor for the CMS experimental chamber* **with a tapering of 2 meters on each side of the structure** *as a function of the maximum frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and the bunch length 7.5 cm.*

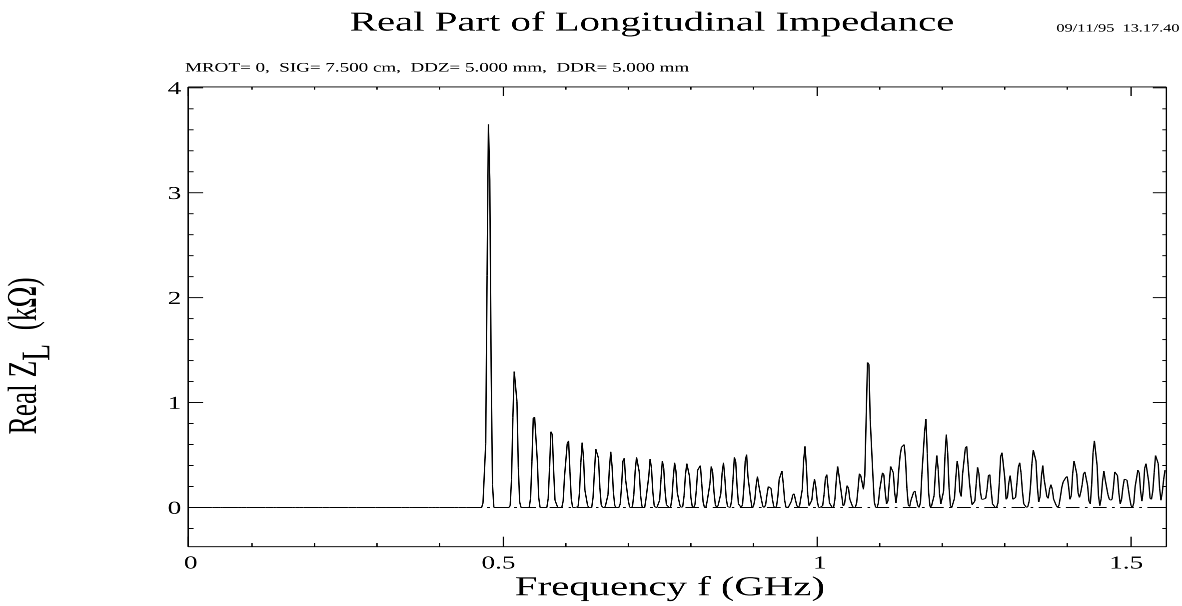

Figure 8: *The real part of the longitudinal impedance for the CMS experimental chamber* **without tapering** *as a function of frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and the bunch length 7.5 cm. The impedance was calculated by evaluating the wake field up to 50 m behind the bunch.*

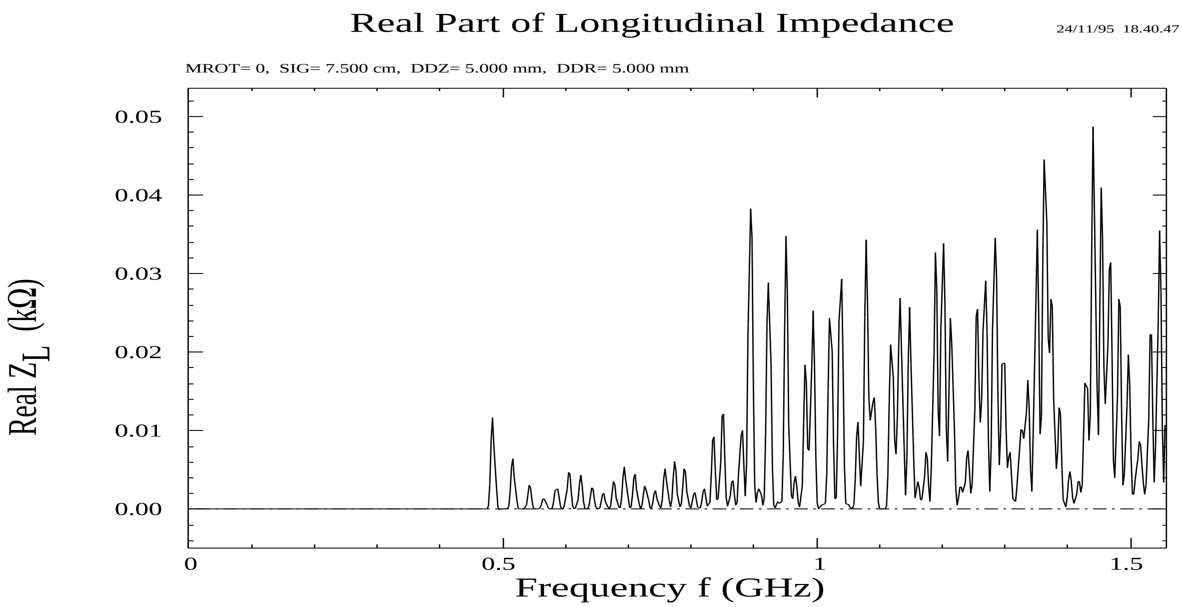

Figure 9: *The real part of the longitudinal impedance for the CMS experimental chamber* **with a tapering of 2 meters on each side** *as a function of frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and the bunch length 7.5 cm. The impedance was calculated by evaluating the wake field up to 50 m behind the bunch.*

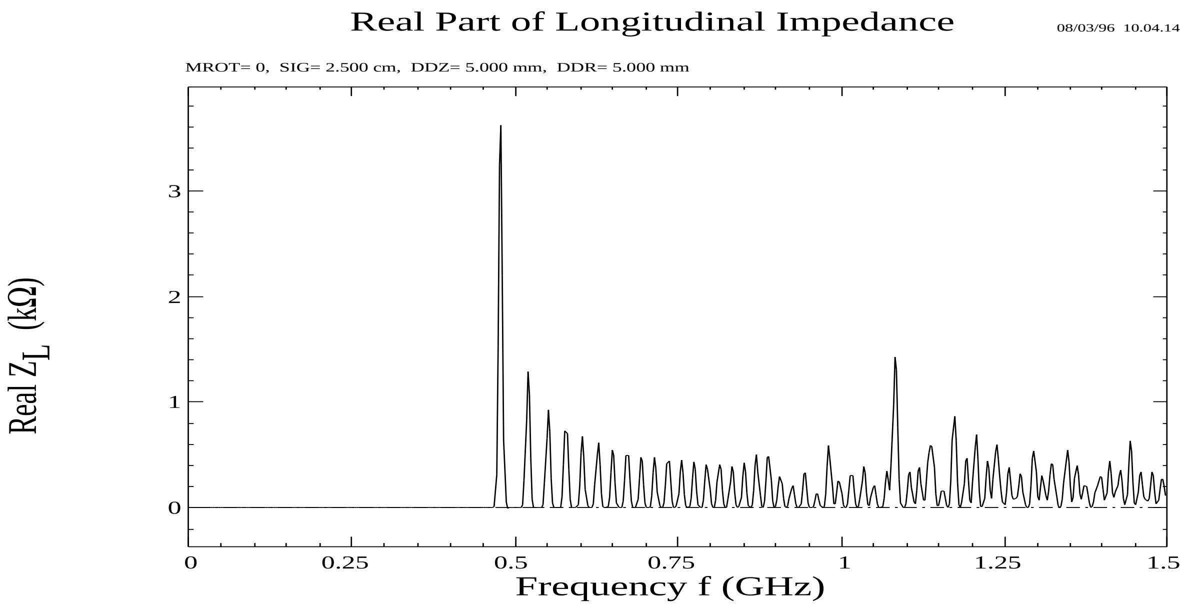

Figure 10: *The real part of the longitudinal impedance for the CMS experimental chamber* **without tapering** *as a function of frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and* **the bunch length 2.5 cm**. *The impedance was calculated by evaluating the wake field up to 50 m behind the bunch.*

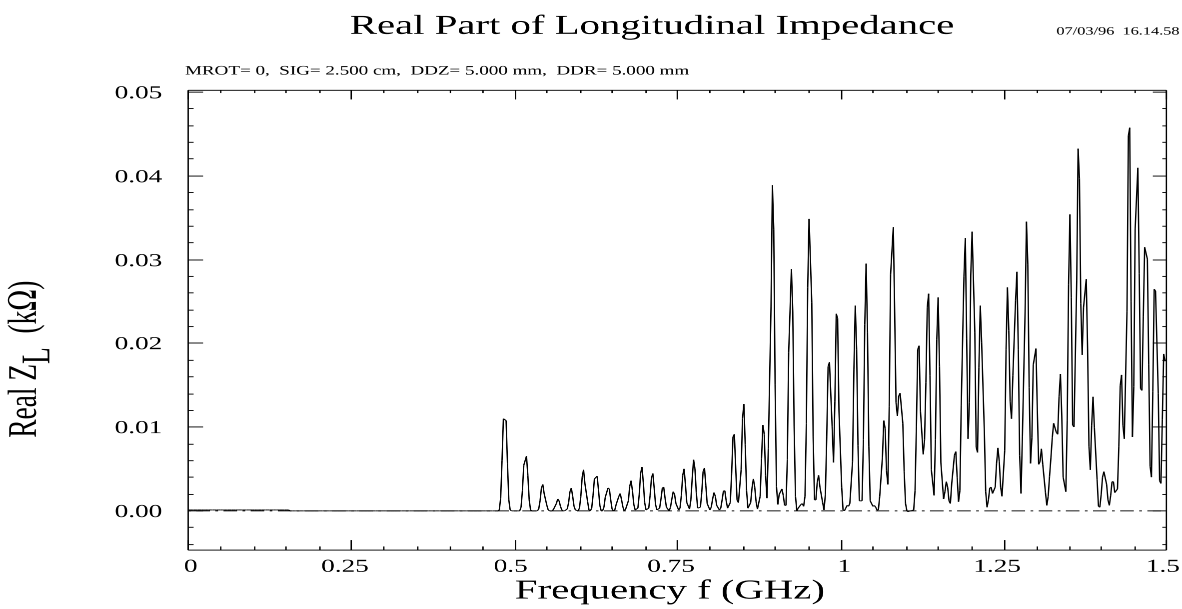

Figure 11: *The real part of the longitudinal impedance for the CMS experimental chamber* **with a tapering of 2 meters on each side** *as a function of frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and* **the bunch length 2.5 cm**. *The impedance was calculated by evaluating the wake field up to 50 m behind the bunch.*

loss factor for the CMS experimental chamber without tapering and Fig. 5 shows the sum of the frequency components of the loss factor spectrum as a function of the maximum frequency. Fig. 6 and Fig. 7 show the the frequency spectrum and the sum of the frequency contributions for the CMS experimental chamber with a tapering of 2 meters on each side of the structure. Fig. 8 and Fig. 9 show the frequency spectrum of the real part of the longitudinal impedance for the chamber without and with tapering respectively. In order to check the validity of the calculations for mode frequencies larger than 1 GHz, the calculations were repeated for a bunch length of 2.5 cm. Fig. 10 and Fig. 11 show the frequency spectrum of the real part of the longitudinal impedance for a bunch length of 2.5 cm for the chamber without and with tapering respectively. In both cases the results are identical and the spectrum above 1 GHz seems to be correct.

A comparison of the longitudinal mode spectrum of the SUPERLANS calculations [1] with the spectra given in Fig. 4 and Fig. 6 shows that the SUPERLANS code does not find all longitudinal modes. While Fig. 4 depicts 9 and Fig. 6 11 lines with frequencies smaller than 700 MHz the SUPERLANS calculations show only 8 lines in both cases (with and without tapering). Because the SUPERLANS program does not find all the trapped modes, we will not rely on it in the following.

Table 1 shows the URMEL data for the first 20 longitudinal modes for the CMS experimental chamber without tapering and Tables 2 and 3 show the corresponding data for a tapering of 2 m on each side of the chamber. While the amplitude of the shunt impedance decreases with increasing mode frequency for the structure without tapering, the shunt impedance increases with increasing mode frequency for the structure with a tapering of $2 \cdot 2$ meter. Tables 2 and 3 show the URMEL data for mode frequencies up to 1 GHz. (The program could not solve for more than 78 modes.) In each case, the modes appear in pairs with equal frequency and quality factor. According to the boundary condition on a surface at the centre of the experimental chamber ($z = 0$) they are labelled (E) and (M). (E) indicates that the tangential electric field on the boundary surface is zero and (M) indicates that the tangential magnetic field is zero. A comparison of the longitudinal mode spectrum of the URMEL calculations with the spectra given in Fig. 4 and Fig. 6 indicates a good agreement between the two calculations. Because the (E) and (M) modes have identical resonance frequencies, they can not be distinguished in the ABCI calculations. Without tapering, the calculation yields 9 frequency lines below 700 MHz and 11 frequencies for a tapering of 2×2 m.

For a given bunch length σ_s, shunt resistance $R_{s,m}$, resonance frequency of the impedance $\omega_{res,m}$ and quality factor Q_m, the longitudinal loss factor per mode (m) can be calculated as [6]

$$k_{L,m} = \frac{R_{s,m}}{S_m} \cdot Re\left[\omega_{1,m} \cdot w(\omega_{1,m} \cdot \sigma_s)\right], \tag{2}$$

with

$$S_m = \sqrt{4Q_m^2 - 1} \qquad \text{and} \qquad \omega_{1,m} = \frac{\omega_{res,m}}{2Q_m} \cdot (i + S_m). \tag{3}$$

$w(z)$ is the complex error function [7]. The total loss factor is given by the sum over all modes

$$k_L = \sum_m k_{L,m}. \tag{4}$$

Choosing a bunch length of 7.5 cm and taking the sum of the first 20 modes we get without tapering for the URMEL data

$$k_L = -0.0866 \text{V/ pC} \tag{5}$$

and with a tapering of 2 m on each side of the structure

$$k_L = -0.001\text{V/ pC}. \tag{6}$$

The first value agrees well with the data in Fig. 5. However, the second value is approximately twice as large as the corresponding value in Fig. 7.

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	Q	$R_s \cdot Q$ [GΩ]
1 (E)	476.3824	42363.9306	4059.5	.8806	58675	51.67
2 (M)	476.3824	42363.9306	4059.5	1.399	58675	82.11
3 (E)	518.8879	46143.8773	3716.22	.7308	69814	51.02
4 (M)	518.8879	46143.8773	3716.22	.1213	69814	8.47
5 (E)	549.9716	48908.1014	3854.47	.3924	71342	27.99
6 (M)	549.9716	48908.1014	3854.47	.2010	71342	14.34
7 (E)	577.3234	51340.4535	4017.78	.4697	71846	33.74
8 (M)	577.3234	51340.4535	4017.78	.0026	71846	.185
9 (E)	602.5603	53584.7310	4182.82	.1486	72028	10.70
10 (M)	602.5603	53584.7310	4182.82	.2515	72028	18.11
11 (E)	626.3810	55703.0680	4345.34	.1218	72075	8.78
12 (M)	626.3810	55703.0680	4345.34	.2305	72075	16.61
13 (E)	649.1642	57729.1418	4504.21	.2316	72062	16.69
14 (M)	649.1642	57729.1418	4504.21	.0889	72062	6.41
15 (E)	671.1407	59683.4771	4659.15	.1117	72024	8.05
16 (M)	671.1407	59683.4771	4659.15	.1935	72024	13.93
17 (E)	692.4582	61579.2085	4809.87	.1164	71983	8.38
18 (M)	692.4582	61579.2085	4809.87	.2012	71983	14.49
19 (E)	713.2111	63424.7310	4956.23	.3009	71951	21.65
20 (M)	713.2111	63424.7310	4956.23	.0103	71951	.742

Table 1: *Parameters for* **the first 20 longitudinal modes** *in the CMS experimental chamber* **without tapering**. *The mode parameters are calculated with* **URMEL** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at* $z = 0$ *in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero.* $\triangle\omega$ *is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	R_s/Q [Ω/m]	Q
1(E)	484.0343	43044.4020	3097.66	.0047	.0605	78129
2(M)	484.0343	43044.4020	3097.66	.0030	.0378	78129
3(E)	516.3655	45919.5643	3327.44	.0001	.0009	77592
4(M)	516.3655	45919.5643	3327.44	.0038	.0487	77592
5(E)	542.7005	48261.4940	3517.50	.0004	.0051	77143
6(M)	542.7005	48261.4940	3517.50	.0018	.0234	77143
7(E)	564.9417	50239.3686	3685.92	.0008	.0100	76635
8(M)	564.9417	50239.3686	3685.92	.0010	.0136	76635
9(E)	584.8385	52008.7594	3824.97	.0012	.0156	76450
10(M)	584.8385	52008.7594	3824.97	.0007	.0093	76450
11(E)	604.4590	53753.5794	3970.02	.0022	.0291	76128
12(M)	604.4590	53753.5794	3970.02	.0023	.0308	76128
13(E)	623.1601	55416.6385	4115.93	.0000	.0001	75701
14(M)	623.1601	55416.6385	4115.93	.0025	.0326	75701
15(E)	641.8311	57077.0209	4238.08	.0012	.0165	75722
16(M)	641.8311	57077.0209	4238.08	.0001	.0012	75722
17(E)	659.2633	58627.2388	4377.06	.0000	.0000	75309
18(M)	659.2633	58627.2388	4377.06	.0008	.0108	75309
19(E)	676.2914	60141.5207	4489.87	.0069	.0910	75313
20(M)	676.2914	60141.5207	4489.87	.0000	.0002	75313
21(E)	692.6507	61596.3273	4626.68	.0136	.1823	74854
22(M)	692.6507	61596.3273	4626.68	.0001	.0012	74854
23(E)	709.7095	63113.3393	4743.73	.0017	.0221	74805
24(M)	709.7095	63113.3393	4743.73	.0009	.0119	74805
25(E)	725.8662	64550.1289	4869.04	.0156	.2095	74539
26(M)	725.8662	64550.1289	4869.04	.0003	.0036	74539
27(E)	741.6272	65951.7297	4993.65	.0044	.0594	74257
28(M)	741.6272	65951.7297	4993.65	.0005	.0068	74257
29(E)	755.4668	67182.4633	5084.24	.0001	.0015	74295
30(M)	755.4668	67182.4633	5084.24	.0022	.0292	74295
31(E)	771.1643	68578.4171	5299.88	.0117	.1607	72753
32(M)	771.1643	68578.4171	5299.88	.0004	.0050	72753
33(E)	787.8777	70064.7132	5218.70	.0168	.2226	75486
34(M)	787.8777	70064.7132	5218.70	.0064	.0851	75486
35(E)	800.7045	71205.3802	5369.10	.0018	.0245	74566

Table 2: *Parameters for* **the first 35 longitudinal modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **URMEL** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	R_s/Q [Ω/m]	Q
36(M)	800.7045	71205.3802	5369.10	.0014	.0186	74566
37(E)	817.7012	72716.8697	5619.55	.0213	.2931	72755
38(M)	817.7012	72716.8697	5619.55	.0002	.0022	72755
39(E)	830.3219	73839.2085	5612.48	.0003	.0037	73971
40(M)	830.3219	73839.2085	5612.48	.0159	.2152	73971
41(E)	845.0495	75148.9106	5812.29	.0005	.0073	72695
42(M)	845.0495	75148.9106	5812.29	.0017	.0231	72695
43(E)	858.1871	76317.2165	5671.64	.0001	.0017	75656
44(M)	858.1871	76317.2165	5671.64	.0296	.3914	75656
45(E)	875.2184	77831.7830	5974.76	.0199	.2724	73243
46(M)	875.2184	77831.7830	5974.76	.0008	.0106	73243
47(E)	886.5461	78839.1374	6038.40	.0000	.0001	73409
48(M)	886.5461	78839.1374	6038.40	.0025	.0344	73409
49(E)	903.1708	80317.5456	6023.87	.0040	.0536	74966
50(M)	903.1708	80317.5456	6023.87	.0030	.0403	74966
51(E)	914.3173	81308.7861	6610.83	.0082	.1183	69153
52(M)	914.3173	81308.7861	6610.83	.0005	.0068	69153
53(E)	926.8294	82421.4673	6130.96	.0093	.1234	75586
54(M)	926.8294	82421.4673	6130.96	.0109	.1443	75586
55(E)	942.6258	83826.2161	6287.44	.0018	.0246	74961
56(M)	942.6258	83826.2161	6287.44	.0106	.1413	74961
57(E)	955.1306	84938.2481	6643.19	.0258	.3592	71888
58(M)	955.1306	84938.2481	6643.19	.0156	.2164	71888
59(E)	969.5535	86220.8537	6368.67	.0017	.0219	76119
60(M)	969.5535	86220.8537	6368.67	.0535	.7024	76119
61(E)	984.5232	87552.0854	6509.67	.0078	.1025	75620
62(M)	984.5232	87552.0854	6509.67	.0376	.4972	75620
63(E)	996.8702	88650.0845	7045.22	.0074	.1053	70748
64(M)	996.8702	88650.0845	7045.22	.0071	.0998	70748
65(E)	1007.9050	89631.3917	7112.15	.0001	.0019	70858
66(M)	1007.9050	89631.3917	7112.15	.0291	.4101	70858
67(E)	1020.4840	90750.0222	6804.32	.0225	.3000	74988
68(M)	1020.4840	90750.0222	6804.32	.0490	.6533	74988
69(E)	1034.6910	92013.4282	6974.76	.0490	.6602	74174
70(M)	1034.6910	92013.4282	6974.76	.0019	.0253	74174

Table 3: *Parameters for* **the next 35 longitudinal modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **URMEL** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

For frequencies larger than 800 MHz, the shunt impedance in the URMEL calculations for the tapered structure becomes very large and can be up to one order of magnitude larger than the shunt impedance of the first trapped mode. Calculating the longitudinal loss factor for the URMEL data of the tapered structure using Equation (2) and considering all modes up to 1.2 GHz, the loss factor becomes

$$k_L = -0.0037\text{V/ pC}, \tag{7}$$

which is almost a factor four larger than the result of the ABCI calculations. Unfortunately, we could neither repeat the URMEL calculations with a finer mesh nor could we calculate any modes beyond 1.2 GHz. In either case, the URMEL program could not find the mode eigenvalues. Therefore, we recalculate the mode spectrum for a finer mesh with the MAFIA program and calculate all modes up to 1.4 GHz. Frequencies above 1.4 GHz are not relevant for a bunch length of 7.5 cm. We used a mesh with 135,000 mesh points and a maximum longitudinal mesh size of 2.4 cm. The smallest mesh size in the radial direction was 2.5 mm. This configuration reached the hardware limit of the workstation. The settings require 100 MB memory and approximately 700 MB disk-space for each symmetry. Table 4 shows the parameters for the first 35 and Tables 5, 6, 7 and 8 the parameters for the first 140 longitudinal modes in the CMS experimental chamber for the tapered and un-tapered structure respectively. For the un-tapered structure, the MAFIA calculations agree rather well with the URMEL data. However, for the tapered vacuum chamber, the shunt impedances of the MAFIA calculations are all ways smaller than the values from the URMEL calculations and the increase of the shunt impedance for mode frequencies larger than 800 MHz is not as strong as in the ABCI and URMEL calculations. Calculating again the longitudinal loss factor for the MAFIA data using Equation (2) and considering all modes up to 1.4 GHz, the loss factor for the tapered structure becomes

$$k_L = -0.0007\text{V/ pC}, \tag{8}$$

which is approximately 30% smaller than the value from the ABCI calculations. Figure 12 shows the sum of the frequency contributions in the URMEL loss factor for the CMS experimental chamber with a tapering of 2 m on each side of the structure as a function of the maximum frequency. Figure 13 shows the corresponding sum of the frequency contributions for the MAFIA calculations.

Even though the loss factor for the MAFIA data agrees much better with the ABCI data than the URMEL calculations, the shunt impedance for mode frequencies larger than 800 MHz does not increase as strong as in the ABCI and URMEL calculations. In order to investigate the validity of the ABCI calculations, we recalculate the loss factor and the frequency spectrum of the real part of the longitudinal impedance in the tapered structure with ABCI and for a bunch length of 2.5 cm. Fig. 14 and 15 show the corresponding real part of the longitudinal impedance and the sum of the frequency contributions in the ABCI loss factor respectively. Both figures confirm the strong increase of the shunt impedance with increasing mode frequency and the resulting discrepancy with the MAFIA calculations still has to be understood. However, a hint for a potential cause of the strong increase of the resonance peaks above 900 MHz in Figures 9 and 14 of the ABCI calculations can be obtained from the electric field patterns in the MAFIA calculations. Figure 16 shows the electric field lines for four different longitudinal modes in the CMS vacuum chamber. In all four cases, the pictures show a blow up of the region near the highest radius of the structure.

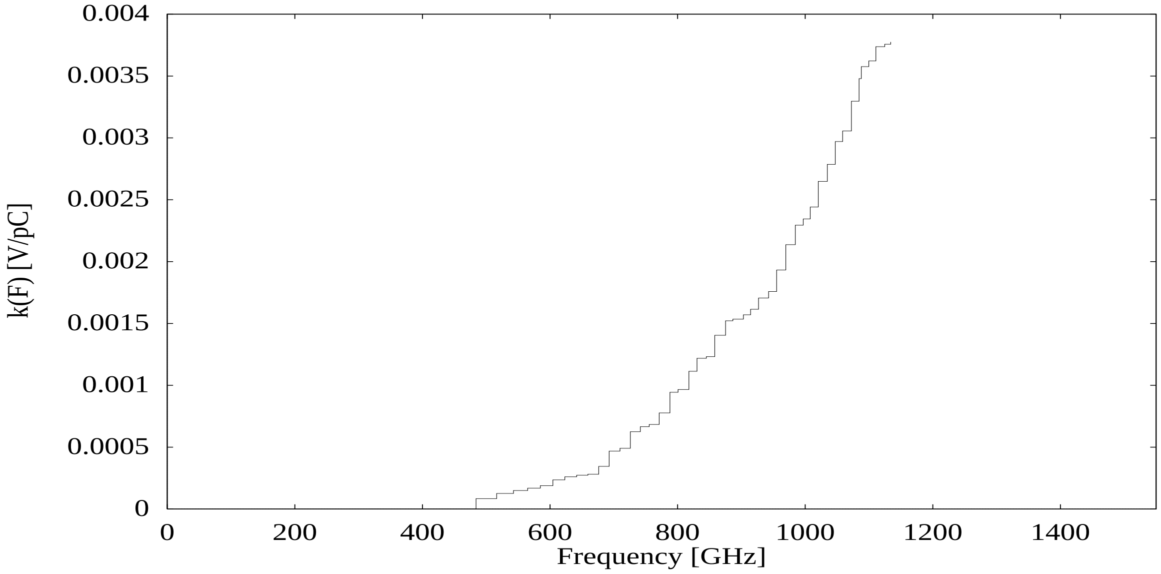

Figure 12: *The sum of the frequency contributions in the* **URMEL** *loss factor for the CMS experimental chamber* **with a tapering of 2 meters on each side of the structure** *as a function of the maximum frequency. The bunch length is 7.5 cm.*

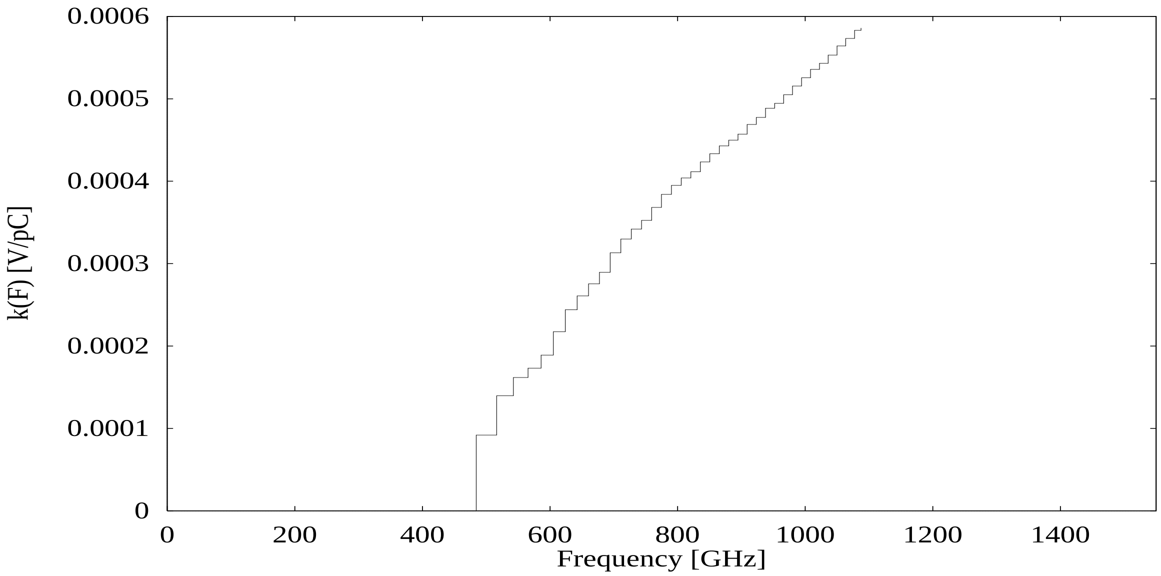

Figure 13: *The sum of the frequency contributions in the* **MAFIA** *loss factor for the CMS experimental chamber* **with a tapering of 2 meters on each side of the structure** *as a function of the maximum frequency. The bunch length is 7.5 cm.*

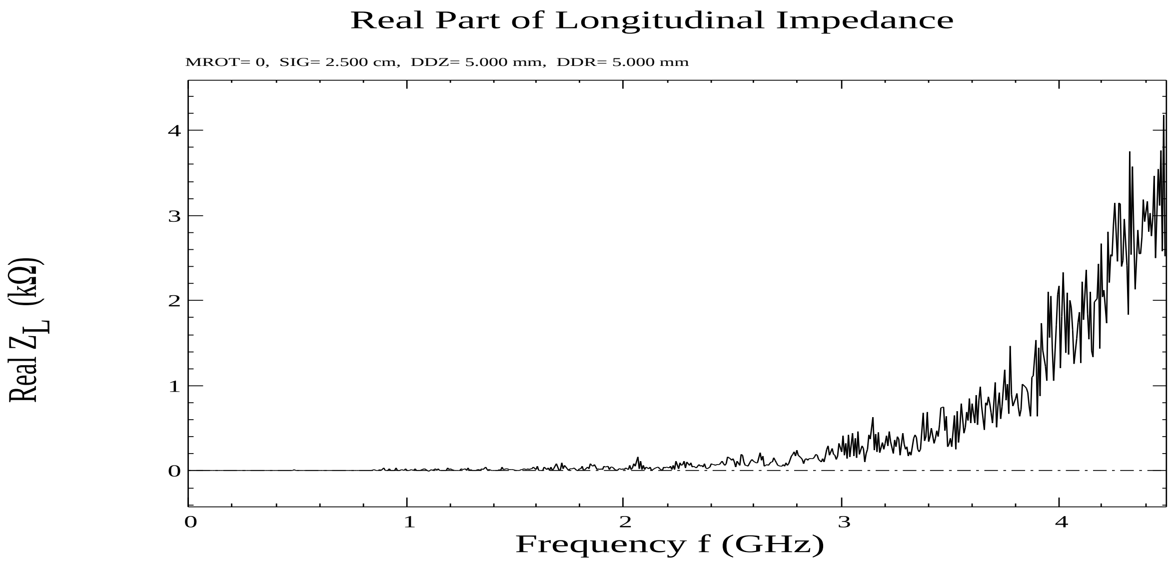

Figure 14: *The real part of the longitudinal impedance for the CMS experimental chamber* **with a tapering of 2 meter on each side of the structure** *as a function of frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and* **the bunch length 2.5 cm**. *The impedance was calculated by evaluating the wake field up to 50 m behind the bunch.*

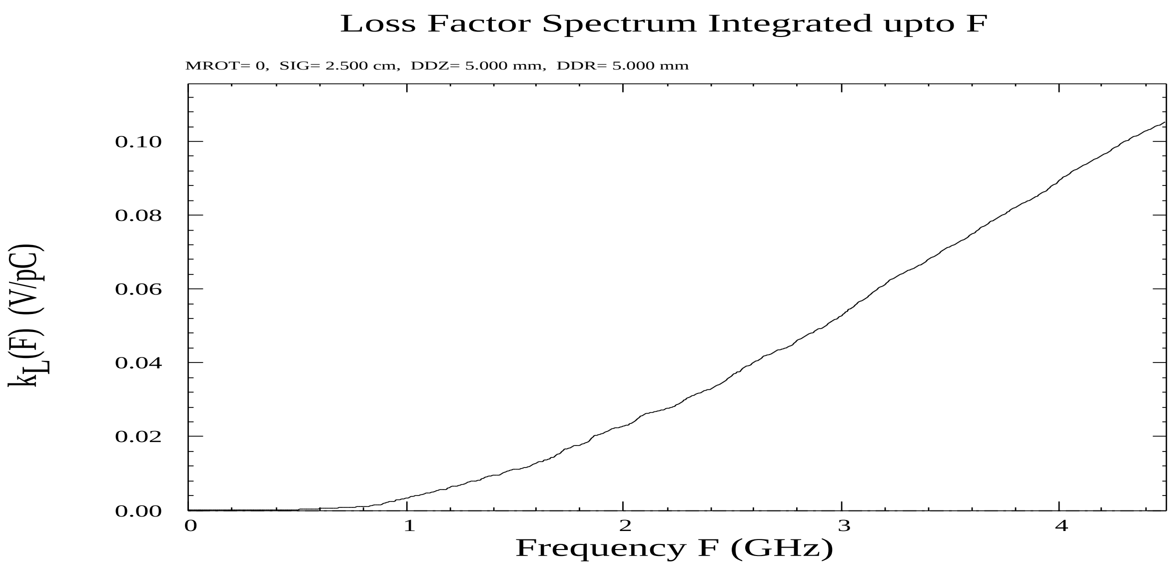

Figure 15: *The sum of the frequency contributions in the ABCI loss factor for the CMS experimental chamber* **with a tapering of 2 meters on each side of the structure** *as a function of the maximum frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and* **the bunch length 2.5 cm**.

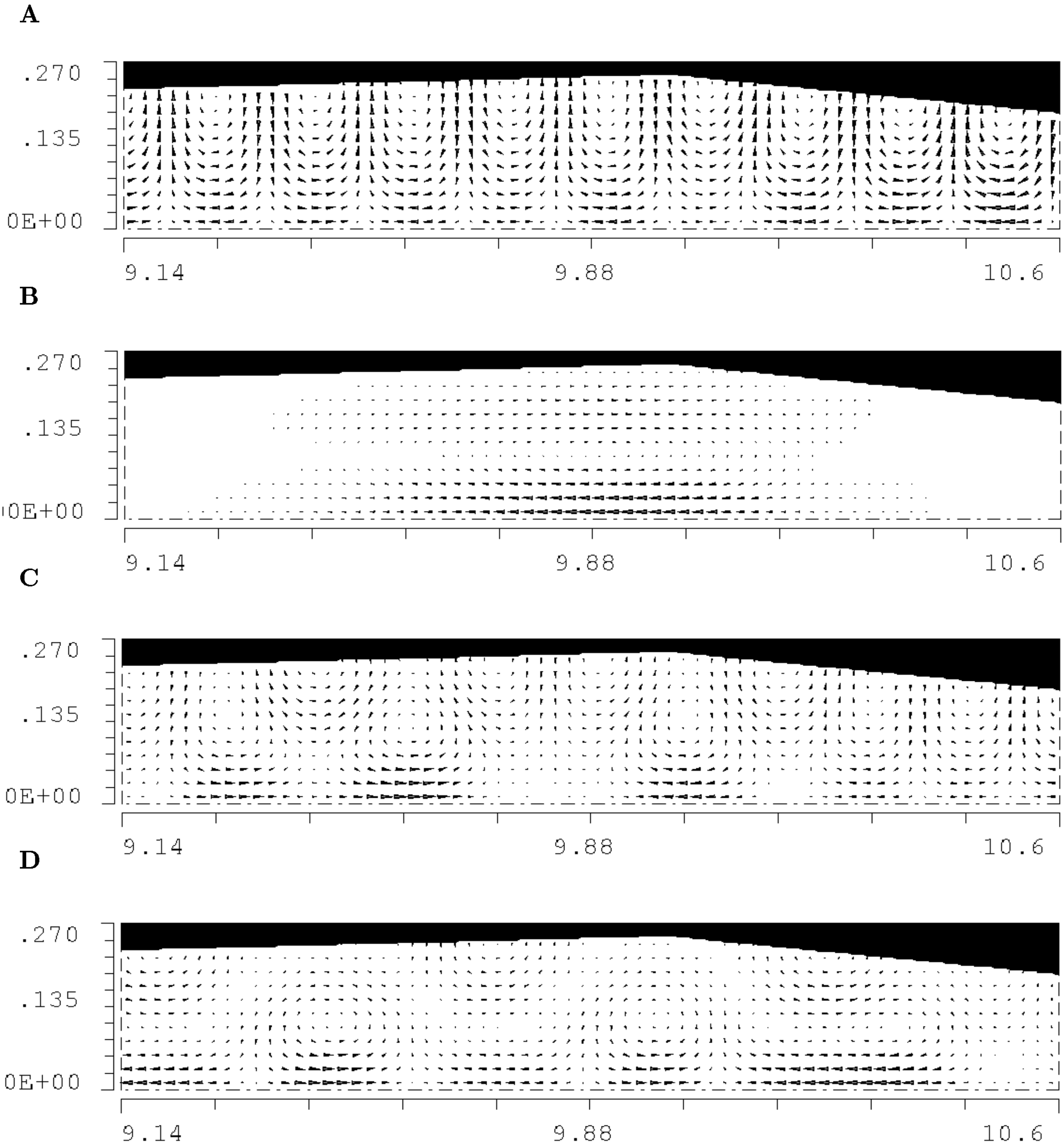

Figure 16: *the electric field lines for four different longitudinal modes in the CMS vacuum chamber. The mode in picture* **A**) *has a resonance frequency of 1.077 GHz and depicts the typical field line distribution for all modes up to 1 GHz. The other three modes have the resonance frequencies* **B**) *1.086 GHz,* **C**) *1.132 GHz, and* **D**) *1.24 GHz. The pictures show a blow up of the region near the highest radius of the structure. The horizontal axis extends from 9.14 m to 10.6 m and the vertical axis from 0 cm to 25 cm. The dark area at the top of each picture corresponds to the wall of the vacuum chamber.*

The horizontal axis extends from 9.14 m to 10.6 m and the vertical axis from 0 cm to 25 cm. The dark area at the top of each picture corresponds to the wall of the vacuum chamber. The first mode has a resonance frequency of 1.077 GHz and depicts the typical field line distribution for all modes up to 1 GHz. The other three modes have the resonance frequencies 1.086 GHz, 1.132 GHz, and 1.24 GHz and are examples for the new kind of trapped modes which appear above 1 GHz. The electric field lines of all three modes is clearly different from the field line pattern of the first mode. These new field line patterns start appearing only for mode frequencies above 1 GHz.

The strong increase of the loss factor for large mode frequencies does not appear for the un-tapered structure. Calculating the longitudinal loss factor for the URMEL and MAFIA data of the un-tapered structure using Equation (2) and considering all modes up to 1.2 GHz the loss factor becomes

$$k_L = -0.107\text{V/ pC}, \tag{9}$$

for the URMEL data and

$$k_L = -0.089\text{V/ pC}, \tag{10}$$

for the MAFIA data. Both values agree within 10% with the ABCI result of $k_L = -0.091\text{V/ pC}$. However, even though the shunt impedance of the tapered structure increases with increasing mode frequency above 800 GHz, the total incoherent loss factor for a bunch length with $\sigma_z \geq 7.5$ cm is still tolerable in all calculations. Only for short bunches with $\sigma_z \ll 7.5$ cm, the incoherent loss factor of the tapered structure becomes as large as the loss factor for the un-tapered vacuum chamber.

While the URMEL program gives good results for the un-tapered structure it seems to over estimate the shunt impedance for the tapered vacuum chamber compared with the ABCI results. Furthermore, the URMEL program did not allow for more than 20,000 mesh points and could not calculate more than 78 trapped modes. Therefore it is hardly possible to check the dependence of the URMEL results on the mesh size. The MAFIA program, on the other hand, can handle up to 150,000 mesh points and 90 trapped modes provided that the system resources are large enough. For 135,000 mesh points the MAFIA program gives good results for the un-tapered structure but underestimates the shunt impedance of the trapped modes by approximately 40% for the tapered structure compared with the ABCI results.

In the following we will use the URMEL and MAFIA data for an analysis of the multi-bunch instability rise times.

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	R_S/Q [Ω]	Q
1(E)	477.8085	42490.7546	4019.05	.2917	4.9069	59443
2(M)	477.8085	42490.7546	4019.05	1.8965	31.9051	59443
3(E)	519.7976	46224.7715	3689.96	.5919	8.4033	70434
4(M)	519.7976	46224.7715	3689.96	.2459	3.4908	70434
5(E)	550.7510	48977.4133	3821.16	.2949	4.0926	72066
6(M)	550.7510	48977.4133	3821.16	.2871	3.9838	72066
7(E)	578.1034	51409.8172	3978.91	.4336	5.9687	72646
8(M)	578.1034	51409.8172	3978.91	.0234	.3224	72646
9(E)	603.1478	53636.9773	4133.41	.1897	2.6004	72960
10(M)	603.1478	53636.9773	4133.41	.2050	2.8096	72960
11(E)	626.8495	55744.7352	4285.74	.0909	1.2436	73132
12(M)	626.8495	55744.7352	4285.74	.2479	3.3901	73132
13(E)	649.6443	57771.8351	4438.92	.2423	3.3116	73176
14(M)	649.6443	57771.8351	4438.92	.0636	.8694	73176
15(E)	671.7950	59741.6644	4584.00	.0772	1.0529	73276
16(M)	671.7950	59741.6644	4584.00	.1972	2.6911	73276
17(E)	693.6052	61681.2120	4737.09	.1527	2.0857	73210
18(M)	693.6052	61681.2120	4737.09	.1028	1.4049	73210
19(E)	714.5411	63543.0058	4891.44	.1692	2.3160	73040
20(M)	714.5411	63543.0058	4891.44	.0645	.8832	73040
21(E)	734.9163	65354.9381	5031.05	.0025	.0341	73038
22(M)	734.9163	65354.9381	5031.05	.2148	2.9412	73038
23(E)	755.1798	67156.9418	5174.66	.1793	2.4573	72969
24(M)	755.1798	67156.9418	5174.66	.0233	.3194	72969
25(E)	774.8018	68901.8900	5315.60	.1322	1.8138	72880
26(M)	774.8018	68901.8900	5315.60	.0656	.9001	72880
27(E)	794.4248	70646.9374	5452.99	.0000	.0000	72843
28(M)	794.4248	70646.9374	5452.99	.1809	2.4838	72843
29(E)	813.5873	72351.0313	5590.51	.1054	1.4482	72765
30(M)	813.5873	72351.0313	5590.51	.0687	.9438	72765
31(E)	832.6285	74044.3338	5720.65	.1671	2.2965	72774
32(M)	832.6285	74044.3338	5720.65	.0045	.0613	72774
33(E)	851.4167	75715.1316	5864.32	.0543	.7480	72593
34(M)	851.4167	75715.1316	5864.32	.1066	1.4681	72593
35(E)	870.1803	77383.7518	5987.62	.0047	.0640	72665

Table 4: *Parameters for* **the first 35 longitudinal modes** *in the CMS experimental chamber* **without tapering**. *The mode parameters are calculated with* **MAFIA** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	R_s/Q [Ω/m]	Q
1(E)	484.3192	43069.7361	3090.11	.0041	.0524	78366
2(M)	484.3192	43069.7361	3090.11	.0043	.0549	78366
3(E)	516.3080	45914.4545	3313.75	.0000	.0004	77904
4(M)	516.3080	45914.4545	3313.75	.0044	.0562	77904
5(E)	542.6364	48255.7909	3499.34	.0004	.0048	77534
6(M)	542.6364	48255.7909	3499.34	.0017	.0217	77534
7(E)	565.5582	50294.1929	3663.37	.0008	.0100	77191
8(M)	565.5582	50294.1929	3663.37	.0003	.0040	77191
9(E)	586.0063	52112.6116	3817.18	.0013	.0165	76759
10(M)	586.0063	52112.6116	3817.18	.0003	.0034	76759
11(E)	605.1722	53817.0000	3957.86	.0012	.0158	76452
12(M)	605.1722	53817.0000	3957.86	.0016	.0206	76452
13(E)	623.9828	55489.8012	4090.62	.0009	.0120	76270
14(M)	623.9828	55489.8012	4090.62	.0018	.0232	76270
15(E)	642.4580	57132.7659	4219.76	.0016	.0207	76125
16(M)	642.4580	57132.7659	4219.76	.0002	.0022	76125
17(E)	660.2875	58718.3182	4345.03	.0002	.0027	75982
18(M)	660.2875	58718.3182	4345.03	.0013	.0177	75982
19(E)	677.4462	60244.2157	4466.82	.0015	.0201	75831
20(M)	677.4462	60244.2157	4466.82	.0000	.0000	75831
21(E)	694.3545	61747.8427	4596.91	.0011	.0142	75524
22(M)	694.3545	61747.8427	4596.91	.0016	.0209	75524
23(E)	711.0208	63229.9514	4721.32	.0001	.0007	75299
24(M)	711.0208	63229.9514	4721.32	.0019	.0250	75299
25(E)	727.4194	64688.2531	4838.95	.0012	.0164	75163
26(M)	727.4194	64688.2531	4838.95	.0002	.0028	75163
27(E)	743.5018	66118.4377	4946.26	.0009	.0122	75158
28(M)	743.5018	66118.4377	4946.26	.0004	.0052	75158
29(E)	759.2199	67516.2168	5073.10	.0000	.0001	74828
30(M)	759.2199	67516.2168	5073.10	.0020	.0269	74828
31(E)	774.6149	68885.2728	5184.21	.0004	.0052	74709
32(M)	774.6149	68885.2728	5184.21	.0017	.0230	74709
33(E)	790.2600	70276.5643	5289.49	.0014	.0193	74701
34(M)	790.2600	70276.5643	5289.49	.0001	.0011	74701
35(E)	805.6390	71644.1948	5404.72	.0012	.0157	74531

Table 5: *Parameters for* **the first 35 longitudinal modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **MAFIA** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at* $z = 0$ *in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero.* $\triangle\omega$ *is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	R_s/Q [Ω/m]	Q
36(M)	805.6390	71644.1948	5404.72	.0001	.0017	74531
37(E)	820.7495	72987.9501	5510.23	.0001	.0009	74475
38(M)	820.7495	72987.9501	5510.23	.0011	.0144	74475
39(E)	835.6099	74309.4604	5618.98	.0001	.0015	74356
40(M)	835.6099	74309.4604	5618.98	.0018	.0237	74356
41(E)	850.4217	75626.6502	5736.25	.0007	.0096	74127
42(M)	850.4217	75626.6502	5736.25	.0009	.0126	74127
43(E)	865.3459	76953.8367	5822.15	.0015	.0206	74315
44(M)	865.3459	76953.8367	5822.15	.0001	.0016	74315
45(E)	880.0602	78262.3554	5946.19	.0012	.0165	74002
46(M)	880.0602	78262.3554	5946.19	.0001	.0007	74002
47(E)	894.6094	79556.1951	6032.19	.0003	.0040	74153
48(M)	894.6094	79556.1951	6032.19	.0011	.0149	74153
49(E)	908.9277	80829.4986	6155.04	.0001	.0010	73836
50(M)	908.9277	80829.4986	6155.04	.0023	.0311	73836
51(E)	923.3296	82110.2381	6244.45	.0002	.0027	73932
52(M)	923.3296	82110.2381	6244.45	.0016	.0222	73932
53(E)	937.7486	83392.4963	6349.52	.0012	.0159	73844
54(M)	937.7486	83392.4963	6349.52	.0014	.0184	73844
55(E)	952.0692	84666.0018	6451.21	.0013	.0176	73790
56(M)	952.0692	84666.0018	6451.21	.0001	.0017	73790
57(E)	966.1563	85918.7473	6544.53	.0023	.0311	73814
58(M)	966.1563	85918.7473	6544.53	.0004	.0053	73814
59(E)	980.1173	87160.2725	6663.29	.0018	.0250	73546
60(M)	980.1173	87160.2725	6663.29	.0011	.0143	73546
61(E)	994.2382	88416.0230	6739.23	.0005	.0062	73765
62(M)	994.2382	88416.0230	6739.23	.0025	.0338	73765
63(E)	1008.2661	89663.4993	6858.95	.0000	.0001	73500
64(M)	1008.2661	89663.4993	6858.95	.0032	.0436	73500
65(E)	1022.3418	90915.2325	6933.86	.0005	.0068	73721
66(M)	1022.3418	90915.2325	6933.86	.0020	.0273	73721
67(E)	1035.9570	92126.0089	7051.74	.0027	.0370	73454
68(M)	1035.9570	92126.0089	7051.74	.0010	.0136	73454
69(E)	1049.8292	93359.6416	7140.82	.0041	.0553	73509
70(M)	1049.8292	93359.6416	7140.82	.0004	.0054	73509

Table 6: *Parameters for* **the next 35 longitudinal modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **MAFIA** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at* $z = 0$ *in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero.* $\triangle\omega$ *is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	R_s/Q [Ω/m]	Q
71(E)	1063.5409	94579.001	7231.73	.0039	.0526	73533
72(M)	1063.5409	94579.001	7231.73	.0001	.0016	73533
73(E)	1077.4448	95815.455	7333.55	.0037	.0507	73460
74(M)	1077.4448	95815.455	7333.55	.0010	.0139	73460
75(E)	1087.4747	96707.396	4596.92	.0001	.0011	118283
76(M)	1087.4747	96707.396	4596.92	.0023	.0196	118283
77(E)	1091.2312	97041.458	7384.46	.0010	.0131	73887
78(M)	1091.2312	97041.458	7384.46	.0020	.0267	73887
79(E)	1104.6775	98237.218	7506.74	.0001	.0013	73579
80(M)	1104.6775	98237.218	7506.74	.0053	.0720	73579
81(E)	1118.2627	99445.329	7607.54	.0000	.0005	73497
82(M)	1118.2627	99445.329	7607.54	.0049	.0670	73497
83(E)	1128.3816	100345.187	4855.01	.0003	.0023	116208
84(M)	1128.3816	100345.187	4855.01	.0016	.0138	116208
85(E)	1131.9957	100666.582	7572.59	.0014	.0189	74743
86(M)	1131.9957	100666.582	7572.59	.0034	.0454	74743
87(E)	1145.6156	101877.773	7809.24	.0032	.0432	73350
88(M)	1145.6156	101877.773	7809.24	.0025	.0338	73350
89(E)	1159.1737	103083.477	7841.90	.0054	.0727	73909
90(M)	1159.1737	103083.477	7841.90	.0002	.0024	73909
91(E)	1161.8330	103319.965	4889.91	.0000	.0003	118799
92(M)	1161.8330	103319.965	4889.91	.0004	.0035	118799
93(E)	1172.5896	104276.528	7977.02	.0070	.0951	73498
94(M)	1172.5896	104276.528	7977.02	.0004	.0051	73498
95(E)	1186.2390	105490.351	8017.95	.0050	.0682	73974
96(M)	1186.2390	105490.351	8017.95	.0015	.0202	73974
97(E)	1190.7451	105891.072	5028.74	.0001	.0010	118394
98(M)	1190.7451	105891.072	5028.74	.0000	.0001	118394
99(E)	1199.7960	106695.950	8131.01	.0019	.0252	73779
100(M)	1199.7960	106695.950	8131.01	.0034	.0463	73779
101(E)	1213.5137	107915.850	8196.87	.0008	.0106	74023
102(M)	1213.5137	107915.850	8196.87	.0058	.0784	74023
103(E)	1216.2885	108162.607	5180.90	.0003	.0029	117382
104(M)	1216.2885	108162.607	5180.90	.0001	.0005	117382
105(E)	1226.8597	109102.687	8323.79	.0001	.0007	73696

Table 7: *Parameters for* **the next 35 longitudinal modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **MAFIA** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at* $z = 0$ *in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero.* $\triangle\omega$ *is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	R_s [MΩ]	R_s/Q [Ω/m]	Q
106(M)	1226.8597	109102.687	8323.79	.0052	.0700	73696
107(E)	1239.5103	110227.679	7039.79	.0015	.0169	88036
108(M)	1239.5103	110227.679	7039.79	.0076	.0866	88036
109(E)	1240.8224	110344.367	6696.72	.0011	.0114	92644
110(M)	1240.8224	110344.367	6696.72	.0026	.0278	92644
111(E)	1253.4113	111463.881	8544.74	.0030	.0412	73344
112(M)	1253.4113	111463.881	8544.74	.0052	.0711	73344
113(E)	1263.4895	112360.119	5460.72	.0000	.0000	115689
114(M)	1263.4895	112360.119	5460.72	.0007	.0060	115689
115(E)	1266.6492	112641.102	8554.05	.0059	.0800	74038
116(M)	1266.6492	112641.102	8554.05	.0019	.0251	74038
117(E)	1280.0734	113834.896	8742.24	.0070	.0958	73212
118(M)	1280.0734	113834.896	8742.24	.0012	.0158	73212
119(E)	1286.2679	114385.758	5495.74	.0001	.0008	117024
120(M)	1286.2679	114385.758	5495.74	.0000	.0001	117024
121(E)	1293.2589	115007.458	8738.00	.0070	.0950	74002
122(M)	1293.2589	115007.458	8738.00	.0001	.0013	74002
123(E)	1306.3093	116168.013	8895.29	.0077	.1047	73427
124(M)	1306.3093	116168.013	8895.29	.0021	.0280	73427
125(E)	1307.8179	116302.167	5618.45	.0000	.0002	116386
126(M)	1307.8179	116302.167	5618.45	.0002	.0018	116386
127(E)	1319.6382	117353.326	8945.73	.0044	.0591	73758
128(M)	1319.6382	117353.326	8945.73	.0044	.0602	73758
129(E)	1328.3992	118132.430	5697.91	.0001	.0004	116569
130(M)	1328.3992	118132.430	5697.91	.0004	.0033	116569
131(E)	1332.6484	118510.311	9106.77	.0018	.0250	73168
132(M)	1332.6484	118510.311	9106.77	.0067	.0916	73168
133(E)	1346.0489	119701.993	9172.89	.0006	.0078	73371
134(M)	1346.0489	119701.993	9172.89	.0109	.1479	73371
135(E)	1348.4406	119914.686	5814.90	.0006	.0050	115947
136(M)	1348.4406	119914.686	5814.90	.0001	.0006	115947
137(E)	1358.9953	120853.294	9221.65	.0000	.0001	73685
138(M)	1358.9953	120853.294	9221.65	.0104	.1412	73685
139(E)	1368.1438	121666.855	5951.56	.0001	.0013	114940
140(M)	1368.1438	121666.855	5951.56	.0005	.0044	114940

Table 8: *Parameters for* **the next 35 longitudinal modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **MAFIA** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

3 Longitudinal Multi-Bunch Instabilities

The trapped modes in the CMS experimental chamber have rather large Q values. In order to estimate the instability rise times, we approximate the impedance of the CMS experimental chamber by a narrow band resonator for each trapped mode. The impedance of a narrow band resonator can be written as

$$Z_0^{||} = \frac{R_s}{1 + iQ \cdot \left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}, \tag{11}$$

where R_s is the shunt resistance, Q the quality factor, and ω_R the resonance frequency of the mode. The growth rate of a longitudinal multi-bunch mode is determined by the interaction matrix $\underline{M}$. The matrix elements are given by [8]

$$M_{k,k'} = i\frac{2\pi N_b N_p r_0 \eta c^2}{\gamma T_0^2 \omega_s^2} l \sum_{-\infty}^{+\infty} \frac{Z_0^{||}(\omega')}{(\omega')} \cdot g_{l,k}(\omega') \cdot g_{l,k'}(\omega'), \tag{12}$$

where $\omega' = (pN_b\omega_0 + \mu\omega_0 + l\omega_s)$. N_p is the number of particles per bunch, N_b the number of bunches in the machine, r_0 the classical proton radius, η the slippage factor, ω_0 the angular revolution frequency, T_0 the revolution time, ω_s the angular synchrotron frequency, and c the speed of light. Table 9 and Table 10 list the corresponding parameters for the LHC.

N_b	N_p	r_0 [A s/ Volt]	η	γ_b	T_0 [s]	$\omega_{s,b}/2\pi$ [Hz]	σ_z [cm]
2835	10^{11}	$1.7 \cdot 10^{-28}$	$3.45 \cdot 10^{-4}$	479.6	$8.8922 \cdot 10^{-5}$	62	13.0

Table 9: *Parameters for the LHC at injection energy. N_b is the number of bunches in the storage ring, N_p the number of particles per bunch, r_0 the classical proton radius, η the slippage factor, γ_b the relativistic gamma, $\omega_{s,b}$ the synchrotron frequency, and σ_z the rms bunch length.*

N_b	N_p	r_0 [A s/ Volt]	η	γ_t	T_0 [s]	$\omega_{s,t}/2\pi$ [Hz]	σ_z [cm]
2835	10^{11}	$1.7 \cdot 10^{-28}$	$3.45 \cdot 10^{-4}$	7460.6	$8.8922 \cdot 10^{-5}$	21	7.5

Table 10: *Parameters for the LHC for luminosity operation. N_b is the number of bunches in the storage ring, N_p the number of particles per bunch, r_0 the classical proton radius, η the slippage factor, γ_t the relativistic gamma, $\omega_{s,t}$ the synchrotron frequency, and σ_z the rms bunch length.*

For a parabolic bunch distribution with rms bunch length $\sigma_z = \hat{z}/\sqrt{5}$, the function $g_{l,k}(\omega')$ is given by

$$g_{l,k}(\omega') = \frac{1}{\hat{z}}\sqrt{\frac{3}{2\pi}\frac{(l+2k+\frac{1}{2})\Gamma(k+\frac{1}{2})\Gamma(l+k+\frac{1}{2})}{k!(l+k)!}} \cdot \frac{J_{l+2k+1/2}(\omega'\hat{z}/c)}{\sqrt{\omega'\hat{z}/c}}. \tag{13}$$

If the problem has been diagonalized, the instability rise time is given by

$$\frac{1}{\tau_{l,n}} = Im(M_{n,n}) \cdot \omega_s. \tag{14}$$

The indices l and n specify the azimuthal and radial mode number respectively. In the following, we will use a numerical routine to diagonalize the interaction matrix.

In order to get a lower estimate for the instability rise time, we take the trapped mode with the largest shunt impedance in Tables 1 and 2 and assume that ω' lies right on the resonance frequency of this mode. For the vacuum chamber without tapering, we find

$$R_s = 0.88\ M\Omega + 1.4\ \mathrm{M\Omega}, \quad \omega_R/2\pi = 476.4\ \mathrm{MHz}, \quad \text{and} \quad Q = 58675 \tag{15}$$

with

$$\tau_{inj} \geq 0.11\ s \qquad \text{and} \qquad \tau_{lum} \geq 0.23\ s \tag{16}$$

for injection and luminosity energy respectively. For the vacuum chamber with a tapering of 2 m on each side of the structure, we find

$$R_s = 0.111\ \mathrm{M\Omega} + 0.006\ \mathrm{M\Omega}, \quad \omega_R/2\pi = 1072.4\ \mathrm{MHz}, \quad \text{and} \quad Q = 76965 \tag{17}$$

with

$$\tau_{inj} \geq 4.2\ s \qquad \text{and} \qquad \tau_{lum} \geq 6.6\ s, \tag{18}$$

which is one order of magnitude larger than the rise times for the un-tapered structure and at a much higher frequency. In the short bunch approximation (bunch length $\ll$ wavelength of the mode), we get

$$\tau_{inj} \geq 0.017\ s \qquad \text{and} \qquad \tau_{lum} \geq 0.27\ s \tag{19}$$

for the vacuum chamber without tapering and

$$\tau_{inj} \geq 0.15\ s \qquad \text{and} \qquad \tau_{lum} \geq 2.3\ s \tag{20}$$

for the vacuum chamber with a tapering of 2 m on each side of the structure.

In all cases, the rise time is large enough that the instability can be damped by a feedback system.

4 Transverse Multi-Bunch Instabilities

For a transverse mode the interaction matrix $\underline{M}$ is given by [8]

$$M_{k,k'} = -i\frac{\pi N_b N_p r_0 c}{\gamma T_0^2 \omega_s \omega_\beta} l \sum_{-\infty}^{+\infty} Z_1^{\perp}(\omega') \cdot g_{l,k}(\omega' - \omega_\xi) \cdot g_{l,k'(\omega' - \omega_\xi)}, \tag{21}$$

where $\omega' = (pN_b\omega_0 + \mu\omega_0 + \omega_\beta + l\omega_s)$ and $\omega_\xi = \xi \cdot \omega_\beta/\eta$. ξ is the chromaticity and η the longitudinal slippage factor. For a bunch with a uniform longitudinal distribution, the function $g_{l,k}$ is given by [8]

$$g_{l,k}(\omega') = \sqrt{\frac{1}{2\pi}\frac{(l+2k+\frac{1}{2})\Gamma(k+\frac{1}{2})\Gamma(l+k+\frac{1}{2})}{k!(l+k)!}} \cdot \frac{J_{l+2k+1/2}(\omega'\hat{z}/c)}{\sqrt{\omega'\hat{z}/c}}. \tag{22}$$

If the problem has been diagonalized, the instability rise time is again given by

$$\frac{1}{\tau_{l,n}} = Im(M_{n,n}) \cdot \omega_s. \tag{23}$$

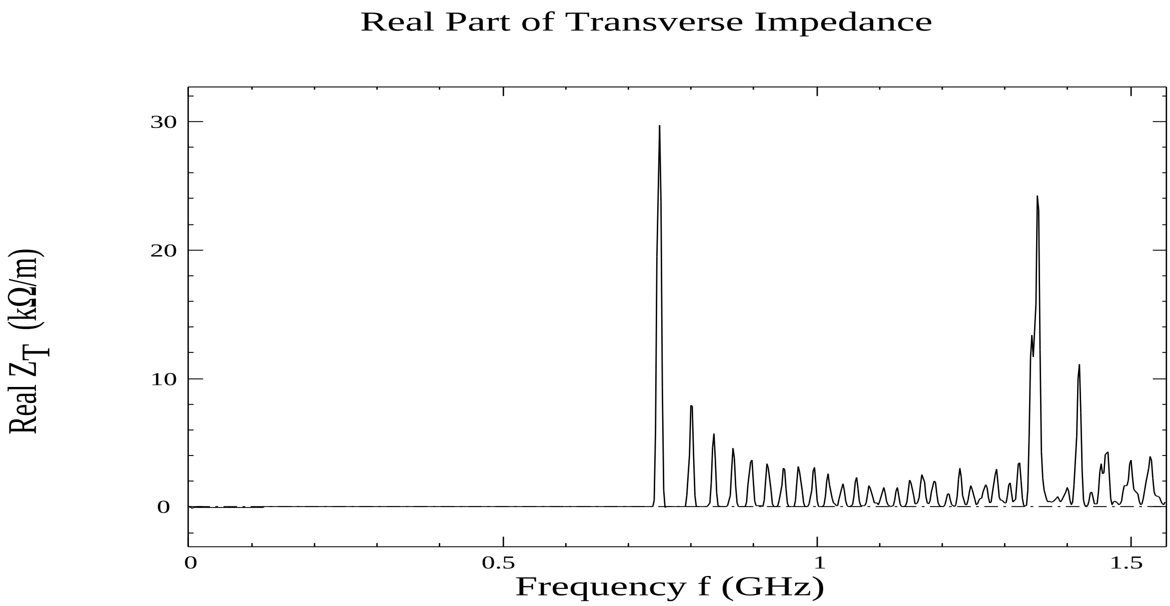

Figure 17: *The real part of the* **transverse impedance** *for the CMS experimental chamber* **without tapering** *as a function of frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and the bunch length 7.5 cm. The impedance was calculated by evaluating the wake field up to 50 m behind the bunch.*

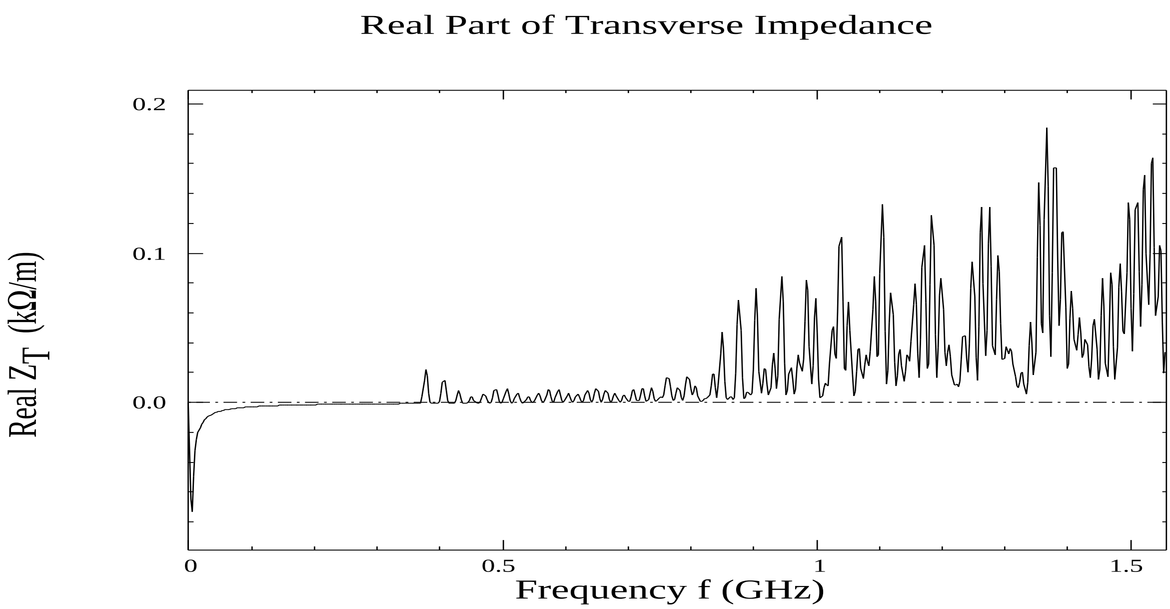

Figure 18: *The real part of the* **transverse impedance** *for the CMS experimental chamber* **with a tapering of** 2×2 **m** *as a function of frequency. The mesh size for the ABCI Calculations is 5 mm in both directions and the bunch length 7.5 cm. The impedance was calculated by evaluating the wake field up to 50 m behind the bunch.*

The transverse impedance of a narrow band resonator can be written as

$$Z_1^{\perp} = \frac{\omega}{c} \cdot \frac{R_s}{1 + iQ \cdot \left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}, \tag{24}$$

where R_s is the shunt resistance, Q the quality factor, and ω_R the resonance frequency of the mode.

In the following, we use a numerical routine for diagonalizing the interaction matrix and calculate the transverse shunt impedance with ABCI and URMEL and MAFIA. Fig. 17 and Fig. 18 show the frequency spectrum of the real part of the transverse impedance for the chamber without and with a tapering of 2×2 m respectively. Again, the impedance seems to increase with increasing mode frequency above 800 MHz for the tapered structure. This interpretation is confirmed by an ABCI calculation with a short bunch of 2.5 cm. Tables 11, 12, 13, 14 and 15, 16 show the parameters for the first 56 transverse modes from the URMEL and MAFIA calculations respectively. (The MAFIA calculations are only done for the tapered structure.) A comparison of the data in Table 11 with the data in Fig. 17 shows that all the modes in Table 11 do not couple to the bunch. Only the modes 37(E) and 38(M), 43(E) and 44(M), and 51(E) and 52(M) in Table 12 appear in Fig.17 and have a significant shunt impedance. For the tapered structure, the transverse impedance of the MAFIA calculation tends to be slightly smaller than in the URMEL calculations and does not show an increase of the shunt impedance for frequencies larger than 600 MHz. Taking the maximum values for the shunt impedance from Tables 11 and 12 and 13 and 14 and inserting the LHC parameters from Tables 9 and 10 into Equations (21) and (23), we can again estimate a lower bound for the instability rise time by assuming that ω' lies right on the resonance frequency of the trapped mode. For the vacuum chamber without tapering, we find

$$Z_{res} = 0.16 \text{ M}\Omega/m + 12.79 \text{ M}\Omega/m, \quad \omega_R/2\pi = 748.5 \text{ MHz}, \quad \text{and} \quad Q = 67798 \tag{25}$$

with

$$\tau_{inj} \geq 1.3 \ s \qquad \text{and} \qquad \tau_{lum} \geq 9.8 \ s \tag{26}$$

for injection and luminosity energy respectively. For the vacuum chamber with a tapering of 2 m on each side of the structure, we find

$$Z_{res} = 0.0288 \text{ M}\Omega/m + 0.0012 \text{ M}\Omega/m, \quad \omega_R/2\pi = 772.2 \text{ MHz}, \quad \text{and} \quad Q = 67952 \tag{27}$$

with

$$\tau_{inj} \geq 550 \ s \qquad \text{and} \qquad \tau_{lum} \geq 4352 \ s. \tag{28}$$

In the short bunch approximation (bunch length $\ll$ wavelength of the mode), we get

$$\tau_{inj} \geq 0.18 \ s \qquad \text{and} \qquad \tau_{lum} \geq 2.7 \ s \tag{29}$$

for the vacuum chamber without tapering and

$$\tau_{inj} \geq 76 \ s \qquad \text{and} \qquad \tau_{lum} \geq 1179 \ s \tag{30}$$

for the vacuum chamber with a tapering of 2 m on each side of the structure. The resonance value of the transverse impedance is related to the shunt resistance by

$$Z^{\perp}_{res} = \frac{\omega_{res}}{c} \cdot R_s. \tag{31}$$

In all cases, the rise time is large enough that the instability can be damped by a feedback system.

5 Summary

The analysis of the multi-bunch instabilities due to the longitudinal and transverse modes in the CMS vacuum chamber showed that the instabilities are not critical for a design with an additional tapering of 2 m on each side of the structure. The analysis assumed a worst case scenario, where the multi-bunch mode frequency lies right on the resonance frequency of the trapped mode in the CMS vacuum chamber. For the tapered structure, the shunt impedance of the longitudinal and transverse trapped modes increases for mode frequencies above 800 MHz. However, for a bunch length of 7.5 cm the rise times of the longitudinal multi-bunch instabilities are not critical. Only for short bunches ($\sigma_s < 7.5$ cm) the rise time for the longitudinal multi-bunch instabilities in the tapered structure can become quite small. In the MAFIA calculations this increase of the longitudinal shunt impedance for frequencies above 800 MHz was much smaller than in the ABCI and URMEL calculations. The transverse multi-bunch instabilities are neither critical for a chamber with or without tapering.

While the URMEL program gives good results for the un-tapered structure is seems to over estimate the shunt impedance for the tapered vacuum chamber compared with the ABCI results. Furthermore, the URMEL program did not allow for more than 20,000 mesh points and could not calculate more than 78 trapped modes. Therfore it is hardly possible to check the dependence of the URMEL results on the mesh size. The MAFIA program, on the other hand, can handel up to 150,000 mesh points and 140 trapped modes provided that the system resources are large enough. For the calculation of 140 transverse modes with 135,000 mesh points, the MAFIA program requires 100 MB memory and approximately 700 MB of disk space for each symmetry. For 135,000 mesh points the MAFIA program gives good results for the un-tapered structure but underestimates the shunt impedance of the trapped modes by approximately 30% for the tapered structure compared with the ABCI results.

So far, we have only analysed the incoherent loss factor. However, because the Q-values of the trapped modes are quite large and because of the large number of bunches in the LHC, the coherent loss factor must also be considered. In this case, the large increase of the shunt impedances for trapped modes with frequencies above 800 MHz will be significant and a corresponding analysis is in work.

Acknowledgements

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Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	$Z^{\perp}_{res}$ [MΩ/m]	R_s/Q [Ω]	Q
1(E)	390.9745	34768.7417	3739.59	.0033	.0076	52275
2(M)	390.9745	34768.7417	3739.59	.0000	.0001	52275
3(E)	422.5297	37574.8955	3947.70	.0028	.0060	53516
4(M)	422.5297	37574.8955	3947.70	.0006	.0012	53516
5(E)	449.4055	39964.9177	4121.40	.0032	.0063	54521
6(M)	449.4055	39964.9177	4121.40	.0002	.0004	54521
7(E)	473.9311	42145.9404	4277.20	.0001	.0002	55402
8(M)	473.9311	42145.9404	4277.20	.0034	.0062	55402
9(E)	496.9788	44195.5358	4421.28	.0033	.0056	56203
10(M)	496.9788	44195.5358	4421.28	.0003	.0005	56203
11(E)	518.9886	46152.8324	4557.41	.0001	.0002	56939
12(M)	518.9886	46152.8324	4557.41	.0033	.0054	56939
13(E)	540.2070	48039.7510	4691.33	.0033	.0051	57575
14(M)	540.2070	48039.7510	4691.33	.0001	.0001	57575
15(E)	560.7375	49865.4958	4836.45	.0011	.0017	57970
16(M)	560.7375	49865.4958	4836.45	.0029	.0042	57970
17(E)	580.6005	51631.8808	4988.23	.0012	.0017	58197
18(M)	580.6005	51631.8808	4988.23	.0022	.0031	58197
19(E)	599.9733	53354.6732	5088.31	.0020	.0028	58956
20(M)	599.9733	53354.6732	5088.31	.0001	.0001	58956
21(E)	619.0860	55054.3353	5151.41	.0015	.0019	60089
22(M)	619.0860	55054.3353	5151.41	.0012	.0016	60089
23(E)	637.8879	56726.3584	5267.10	.0001	.0002	60554
24(M)	637.8879	56726.3584	5267.10	.0007	.0008	60554
25(E)	656.4413	58376.2828	5373.97	.0051	.0061	61076
26(M)	656.4413	58376.2828	5373.97	.0000	.0000	61076
27(E)	674.6648	59996.8697	5450.16	.0183	.0209	61894
28(M)	674.6648	59996.8697	5450.16	.0048	.0055	61894

Table 11: *Parameters for* **the first 28 transverse modes** *in the CMS experimental chamber* **without tapering**. *The mode parameters are calculated with* **URMEL** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	$Z^{\perp}_{res}$ [MΩ/m]	R_s/Q [Ω]	Q
29(E)	692.8157	61611.0004	5557.29	.0488	.0540	62334
30(M)	692.8157	61611.0004	5557.29	.0000	.0000	62334
31(E)	710.8437	63214.2019	5657.69	.0006	.0006	62821
32(M)	710.8437	63214.2019	5657.69	.0027	.0029	62821
33(E)	728.7317	64804.9533	5695.35	.0041	.0042	63976
34(M)	728.7317	64804.9533	5695.35	.0463	.0474	63976
35(E)	746.3029	66367.5322	5738.67	.0023	.0023	65024
36(M)	746.3029	66367.5322	5738.67	.0302	.0297	65024
37(E)	748.5056	66563.4149	5520.12	.1629	.1532	67798
38(M)	748.5056	66563.4149	5520.12	12.7882	12.0236	67798
39(E)	763.7360	67917.8301	5776.95	.0067	.0063	66102
40(M)	763.7360	67917.8301	5776.95	.0362	.0342	66102
41(E)	781.8255	69526.5007	5811.88	.0127	.0115	67261
42(M)	781.8255	69526.5007	5811.88	.0095	.0086	67261
43(E)	799.1191	71064.3931	4819.84	2.7250	1.9626	82899
44(M)	799.1191	71064.3931	4819.84	.7790	.5611	82899
45(E)	799.4850	71096.9320	5604.05	.6264	.5241	71331
46(M)	799.4850	71096.9320	5604.05	.1056	.0884	71331
47(E)	817.0486	72658.8350	6071.37	.0029	.0025	67287
48(M)	817.0486	72658.8350	6071.37	.0274	.0238	67287
49(E)	834.1529	74179.8933	5998.86	.1272	.1046	69526
50(M)	834.1553	74180.1067	5997.84	.0405	.0333	69538
51(E)	834.8271	74239.8488	4755.98	1.7090	1.1129	87766
52(M)	834.8242	74239.5909	4743.75	1.2751	.8282	87992
53(E)	850.5077	75634.2997	6640.23	.0202	.0177	64042
54(M)	850.5005	75633.6594	6635.20	.0213	.0187	64090
55(E)	865.1858	76939.5998	5561.82	.0449	.0319	77779
56(M)	864.9356	76917.3499	5652.58	.2078	.1498	76508

Table 12: *Parameters for* **the next 28 transverse modes** *in the CMS experimental chamber* **without tapering**. *The mode parameters are calculated with* **URMEL** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	$Z^{\perp}_{res}$ [MΩ/m]	R_{res}/Q [Ω]	Q
1(E)	377.0318	33528.8395	3609.34	.0009	.0021	52230
2(M)	377.0318	33528.8395	3609.34	.0135	.0327	52230
3(E)	405.9037	36096.3717	3778.17	.0013	.0029	53717
4(M)	405.9037	36096.3717	3778.17	.0081	.0177	53717
5(E)	429.3622	38182.4989	3925.92	.0007	.0014	54683
6(M)	429.3622	38182.4989	3925.92	.0022	.0045	54683
7(E)	449.9036	40009.2130	4036.02	.0029	.0054	55736
8(M)	449.9036	40009.2130	4036.02	.0002	.0003	55736
9(E)	469.4352	41746.1272	4138.69	.0001	.0002	56713
10(M)	469.4352	41746.1272	4138.69	.0036	.0064	56713
11(E)	487.8627	43384.8555	4265.50	.0061	.0105	57187
12(M)	487.8627	43384.8555	4265.50	.0005	.0008	57187
13(E)	505.6887	44970.0934	4354.28	.0017	.0027	58068
14(M)	505.6887	44970.0934	4354.28	.0025	.0041	58068
15(E)	522.8752	46498.4615	4460.94	.0007	.0011	58606
16(M)	522.8752	46498.4615	4460.94	.0031	.0048	58606
17(E)	539.6390	47989.2397	4550.00	.0019	.0029	59301
18(M)	539.6390	47989.2397	4550.00	.0000	.0000	59301
19(E)	555.9085	49436.0605	4667.11	.0008	.0012	59556
20(M)	555.9085	49436.0605	4667.11	.0046	.0066	59556
21(E)	571.7696	50846.5629	4782.52	.0004	.0005	59777
22(M)	571.7696	50846.5629	4782.52	.0071	.0099	59777
23(E)	587.1030	52210.1378	4889.84	.0024	.0032	60033
24(M)	587.1030	52210.1378	4889.84	.0004	.0005	60033
25(E)	602.1747	53550.4402	4952.09	.0028	.0036	60800
26(M)	602.1747	53550.4402	4952.09	.0000	.0000	60800
27(E)	617.1860	54885.3713	4991.88	.0010	.0012	61819
28(M)	617.1860	54885.3713	4991.88	.0018	.0022	61819

Table 13: *Parameters for* **the first 28 transverse modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **URMEL** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	$Z^{\perp}_{res}$ [MΩ/m]	R_{res}/Q Ω	Q
29(E)	632.0662	56208.6438	5088.44	.0017	.0020	62108
30(M)	632.0662	56208.6438	5088.44	.0087	.0106	62108
31(E)	646.6402	57504.6865	5188.06	.0000	.0001	62320
32(M)	646.6402	57504.6865	5188.06	.0186	.0220	62320
33(E)	660.9645	58778.5238	5247.92	.0000	.0000	62974
34(M)	660.9645	58778.5238	5247.92	.0014	.0017	62974
35(E)	675.1268	60037.9546	5288.48	.0094	.0104	63830
36(M)	675.1268	60037.9546	5288.48	.0044	.0048	63830
37(E)	689.3647	61304.1085	5396.20	.0255	.0276	63875
38(M)	689.3647	61304.1085	5396.20	.0067	.0072	63875
39(E)	703.4898	62560.2312	5479.66	.0001	.0001	64191
40(M)	703.4898	62560.2312	5479.66	.0117	.0123	64191
41(E)	717.0502	63766.1361	5478.52	.0245	.0249	65442
42(M)	717.0502	63766.1361	5478.52	.0012	.0012	65442
43(E)	730.9768	65004.6065	5583.39	.0092	.0092	65460
44(M)	730.9768	65004.6065	5583.39	.0229	.0229	65460
45(E)	744.7866	66232.6901	5596.79	.0020	.0019	66537
46(M)	744.7866	66232.6901	5596.79	.0054	.0052	66537
47(E)	758.4433	67447.1587	5516.27	.0021	.0019	68746
48(M)	758.4371	67446.6074	5518.23	.0065	.0060	68721
49(E)	760.8254	67658.9951	3921.17	.0020	.0013	97015
50(M)	760.8246	67658.9240	3920.56	.0119	.0077	97030
51(E)	772.2423	68674.2819	5682.26	.0288	.0262	67952
52(M)	772.2490	68674.8777	5679.22	.0012	.0011	67989
53(E)	786.2398	69919.0574	5634.11	.0032	.0028	69775
54(M)	786.2016	69915.6603	5626.17	.0080	.0069	69870
55(E)	.0000	.0000	NaN	.0000	.0000	
56(M)	796.7199	70851.0360	4376.14	.0034	.0023	91030

Table 14: *Parameters for* **the next 28 transverse modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **URMEL** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at* $z = 0$ *in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero.* $\triangle\omega$ *is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	$Z^{\perp}_{res}$ [MΩ/m]	R_s/Q [Ω]	Q
1(E)	377.1658	33540.7599	3556.69	.0012	.0028	53022
2(M)	377.1658	33540.7599	3556.69	.0122	.0290	53022
3(E)	405.7546	36083.1144	3745.47	.0006	.0014	54166
4(M)	405.7546	36083.1144	3745.47	.0083	.0179	54166
5(E)	429.4033	38186.1523	3904.87	.0014	.0028	54983
6(M)	429.4033	38186.1523	3904.87	.0031	.0063	54983
7(E)	450.3904	40052.5050	4051.15	.0023	.0043	55588
8(M)	450.3904	40052.5050	4051.15	.0000	.0000	55588
9(E)	469.6847	41768.3132	4171.86	.0001	.0003	56292
10(M)	469.6847	41768.3132	4171.86	.0028	.0050	56292
11(E)	488.0729	43403.5465	4269.43	.0043	.0074	57159
12(M)	488.0729	43403.5465	4269.43	.0001	.0001	57159
13(E)	505.8393	44983.4878	4371.61	.0018	.0029	57855
14(M)	505.8393	44983.4878	4371.61	.0032	.0053	57855
15(E)	523.1554	46523.3766	4471.80	.0008	.0012	58495
16(M)	523.1554	46523.3766	4471.80	.0025	.0039	58495
17(E)	540.0052	48021.8078	4583.46	.0019	.0028	58908
18(M)	540.0052	48021.8078	4583.46	.0001	.0001	58908
19(E)	556.3771	49477.7313	4698.26	.0005	.0007	59211
20(M)	556.3771	49477.7313	4698.26	.0025	.0036	59211
21(E)	572.3834	50901.1431	4786.85	.0006	.0008	59787
22(M)	572.3834	50901.1431	4786.85	.0031	.0043	59787
23(E)	588.1660	52304.6692	4877.08	.0024	.0033	60299
24(M)	588.1660	52304.6692	4877.08	.0010	.0013	60299
25(E)	603.6331	53680.1377	4982.20	.0026	.0034	60579
26(M)	603.6331	53680.1377	4982.20	.0001	.0002	60579
27(E)	618.9285	55040.3266	5066.21	.0006	.0007	61084
28(M)	618.9285	55040.3266	5066.21	.0017	.0021	61084
29(E)	634.0739	56387.1857	5159.60	.0002	.0002	61446
30(M)	634.0739	56387.1857	5159.60	.0031	.0038	61446
31(E)	649.0557	57719.4959	5243.20	.0019	.0022	61895
32(M)	649.0557	57719.4959	5243.20	.0017	.0020	61895
33(E)	663.8183	59032.3048	5329.22	.0033	.0039	62281

Table 15: *Parameters for* **the first 33 transverse modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **MAFIA** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at* $z = 0$ *in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero.* $\triangle\omega$ *is the resonance width of the trapped mode.*

Mode #	$\omega_R/2\pi$ [MHz]	ω_R/ω_0	$\triangle\omega/2\pi$ [KHz]	$Z^{\perp}_{res}$ [MΩ/m]	R_s/Q [Ω]	Q
34(M)	663.8183	59032.3048	5329.22	.0002	.0003	62281
35(E)	678.4817	60336.2976	5418.66	.0022	.0025	62606
36(M)	678.4817	60336.2976	5418.66	.0003	.0004	62606
37(E)	692.9968	61627.1034	5499.63	.0007	.0008	63004
38(M)	692.9968	61627.1034	5499.63	.0020	.0022	63004
39(E)	707.5115	62917.8743	5586.79	.0000	.0000	63320
40(M)	707.5115	62917.8743	5586.79	.0031	.0034	63320
41(E)	721.8396	64192.0532	5657.49	.0008	.0008	63795
42(M)	721.8396	64192.0532	5657.49	.0031	.0032	63795
43(E)	736.0572	65456.3996	5748.92	.0018	.0018	64017
44(M)	736.0572	65456.3996	5748.92	.0011	.0011	64017
45(E)	750.1448	66709.1819	5811.38	.0022	.0022	64541
46(M)	750.1448	66709.1819	5811.38	.0001	.0001	64541
47(E)	761.1866	67691.1201	3922.51	.0020	.0013	97028
48(M)	761.1866	67691.1201	3922.51	.0031	.0020	97028
49(E)	764.3932	67976.2749	5820.76	.0034	.0033	65661
50(M)	764.3932	67976.2749	5820.76	.0005	.0004	65661
51(E)	778.3623	69218.5221	5977.84	.0014	.0013	65104
52(M)	778.3623	69218.5221	5977.84	.0020	.0019	65104
53(E)	792.2509	70453.6117	6053.63	.0003	.0003	65436
54(M)	792.2509	70453.6117	6053.63	.0031	.0028	65436
55(E)	797.8597	70952.3949	4036.44	.0012	.0007	98832
56(M)	797.8597	70952.3949	4036.44	.0018	.0011	98832
57(E)	806.1787	71692.1943	6133.62	.0000	.0000	65718
58(M)	806.1787	71692.1943	6133.62	.0032	.0029	65718
59(E)	819.9148	72913.7253	6194.49	.0006	.0005	66181
60(M)	819.9148	72913.7253	6194.49	.0019	.0017	66181
61(E)	827.9348	73626.9281	4210.15	.0001	.0001	98326
62(M)	827.9348	73626.9281	4210.15	.0010	.0006	98326
63(E)	833.7360	74142.8171	6280.78	.0019	.0017	66372
64(M)	833.7360	74142.8171	6280.78	.0011	.0010	66372
65(E)	847.4641	75363.6397	6335.14	.0030	.0025	66886
66(M)	847.4641	75363.6397	6335.14	.0002	.0002	66886

Table 16: *Parameters for* **the next 33 transverse modes** *in the CMS experimental chamber* **with a tapering of 2 m on each side of the structure**. *The mode parameters are calculated with* **MAFIA** *and the (E) and (M) labels in the mode number column specify the boundary condition on a surface at $z = 0$ in the experimental chamber: (E) = tangential electric field is zero, (M) = tangential magnetic field is zero. $\triangle\omega$ is the resonance width of the trapped mode.*

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