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Transverse Impedance of Ferrite Elements

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Summary

A specific feature of ferrites is that these materials behave either as metals or magneto-dielectrics, depending on the frequency range. Their magnetic permeability is a function of frequency as well. In this paper, the transverse impedance of a ferrite kicker is calculated. The method suggested in Ref. [1] is generalized here for ferrites. Namely, in [1] it is assumed that the electric field of the beam charge dipole is always perfectly shielded. In fact, this assumption requires the conductivity being high compared with the frequency. This is not necessarily true for the high frequencies of a proton single-bunch spectrum. That is why the dynamics of the electric shielding has to be taken into account for ferrite kickers. The generalized analytic result is applied for the ferrite MKE kickers at the CERN SPS, and a fairly good agreement with observations [2] is found.

1. Finite Time of Electric Shielding

An essential methodical point of Ref. [1] is a distinction between the fields excited by the beam electric dipole and the beam magnetic dipole. Due to the superposition principle, it always can be done, and the benefit of this separation is a significant simplification of the calculations without any loss of generality. For metals, the response on the beam electric dipole is an instantaneous shielding of its fields by charges induced at the very surface of the chamber. However, the model of instantaneous electric shielding, accepted in [1], might not be as good for ferrites, whose resistivity is many orders of magnitude higher, than that of metals. For ferrites, the current induced by the direct beam fields might be so small that the shielding time is comparable or even longer than the bunch time of flight. In this case, the idea of instantaneous electric shielding is not applicable. The purpose of this paper is to modify the results of Ref. [1] for the case of an arbitrary conductivity. To make the main idea clear, it is done here for the simple case of infinitely thick ferrite open for the beam, and round symmetry is assumed.

The scalar potential directly associated with the beam charge dipole is

$$\Phi(r,\theta,t) = \frac{2\lambda x(t)}{r}\cos\theta, \qquad (1)$$

where λ is the beam linear density and $x(t) = x_0\, e^{-i\omega t}$ is the beam offset. The total field is the sum of the direct beam field (1) and the field from induced surface charges.

The surface density of the induced charges can be presented as

$$\Sigma(\theta,t)=\frac{\lambda x_0}{\pi a^2}\cos\theta\,\xi(t)\,, \tag{2}$$

where $\xi(t)=\xi_0\, e^{-i\omega t}$ is the shielding parameter (for perfect shielding $\xi_0=1$).

Due to incomplete shielding, there is an electric field inside the medium, whose radial component creates a radial current $j_r=\sigma E_r$, which changes the surface density. Taking into account that

$$\frac{\partial\Sigma}{\partial t}=-j_r=-\sigma E_r=\sigma\frac{2\lambda x_0 e^{-i\omega t}}{a^2}\left(1-\xi_0\right)\cos\theta\,, \tag{3}$$

leads to

$$\dot{\xi}=2\pi\sigma\left(1-\xi_0\right)e^{-i\omega t}\,,\quad \text{or}\quad \xi_0=\frac{1}{1-\dfrac{i\omega}{2\pi\sigma}}\,. \tag{4}$$

The last formula gives the result sought for. For metals, the conductivity is so high, $\sigma\geq 10^{16}\ \mathrm{s}^{-1}$, that electric shielding is absolute for any frequency of interest. For ferrites, the situation can be different. For example, the 4A4 ferrite is specified by the manufacturer by the resistivity $\rho=30\,\Omega\mathrm{m}$ [3], corresponding to a conductivity $\sigma=3\cdot 10^{8}\ \mathrm{s}^{-1}$, which lies in the range of single-bunch frequencies for 4 ns total bunch length.

If a ferrite is an only finite-thickness material layer in the chamber, then the finite time of electric compensation results in an additional term to the transverse impedance

$$Z_\perp^\xi=i\frac{Z_0}{2\pi a^2\beta}(1-\xi_0)=i\frac{Z_0}{2\pi a^2\beta}\frac{1}{1+2\pi i\sigma/\omega}\,. \tag{5}$$

Up to this point, it was assumed for simplicity that a real part of the material dielectric function is the same as for vacuum, $\varepsilon'\equiv\mathrm{Re}(\varepsilon)=1$. In general, it is not so, and Eq. (5) has to be corrected. An easy way to do this is to notice that any solution of the Maxwell equations is an analytical function of the complex dielectric permittivity $\varepsilon=\varepsilon'+4\pi i\sigma/\omega$. Thus, Eq. (5) has to be presented as an analytical function of ε, and this form of the result will be automatically valid for any real part ε'. This analytical continuation is unique and presented below

$$Z_\perp^\xi=i\frac{Z_0}{\pi a^2\beta}\frac{1}{1+\varepsilon}=i\frac{Z_0}{\pi a^2\beta}\frac{1}{1+\varepsilon'+4\pi i\sigma/\omega}\,. \tag{5a}$$

The total transverse impedance of Ref. [1] (see Eq. (13)), can be written

$$Z_\perp=Z_\perp^\sigma+Z_\perp^\infty+Z_\perp^\xi\,, \tag{6}$$

where $Z_\perp^\infty=-i Z_0/(2\pi a^2\beta\gamma^2)$ is the conventional space-charge term, and the resistive impedance $Z_\perp^\sigma$ is found in Ref. [1] from the boundary conditions for the vector potential driven by the beam magnetic dipole.

Note that the correction $Z_\perp^\xi$ is required only if the ferrite is not coated. Even a tiny metallic coating leads to an instantaneous induction of the shielding charge density on the inner metallic surface. Thus, there is no correction to the electric contribution in this case.

2. SPS Kicker

The method described is applied here for calculation of the transverse impedance for the SPS MKE kicker. For purposes of this paper, the kicker can be represented by a pair of identical ferrite plates. The half-gap is $a = 16\ \mathrm{mm}$, the plate thickness is $d = 60\ \mathrm{mm}$, the length of a single kicker in the longitudinal direction is $L = 1.66\ \mathrm{m}$, and the horizontal width is about an order of magnitude larger than the half-gap, so it is taken as infinite. The 8C11 ferrite parameters are assumed according to data presented to the author by E. Gaxiola [4]. The magnetic permeability of this ferrite is presented in Fig. 1.

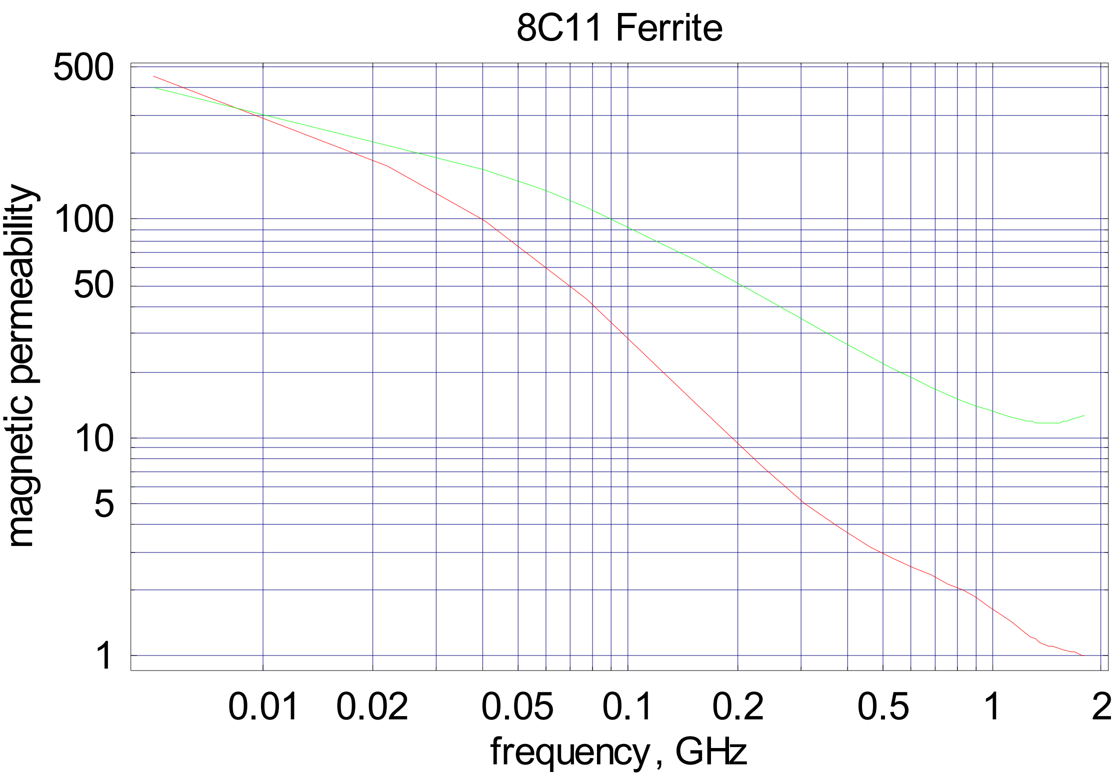

Fig. 1: Magnetic permeability of 8C11 ferrite [3]: real (red) and imaginary (green) parts.

The real part of the dielectric permittivity is fairly constant in the interesting frequency range $10\,\mathrm{MHz} < f < 2\,\mathrm{GHz}$ and amounts to $\varepsilon' = 12$, while the resistivity is estimated as $\rho \geq 10^5\,\Omega\mathrm{m}$ [3].

Fields, excited in the ferrite by the beam magnetic dipole, decay in the radial direction with the skin depth $\delta = 1/\mathrm{Re}[\kappa]$, where the complex wave number κ is determined by the material properties at the given frequency,

$$\kappa^2 = -\frac{\omega^2 \mu \varepsilon}{c^2}, \qquad \varepsilon = \varepsilon' + \frac{4\pi i \sigma}{\omega}. \tag{7}$$

A plot of the skin depth as a function of frequency is shown in Fig. 2.

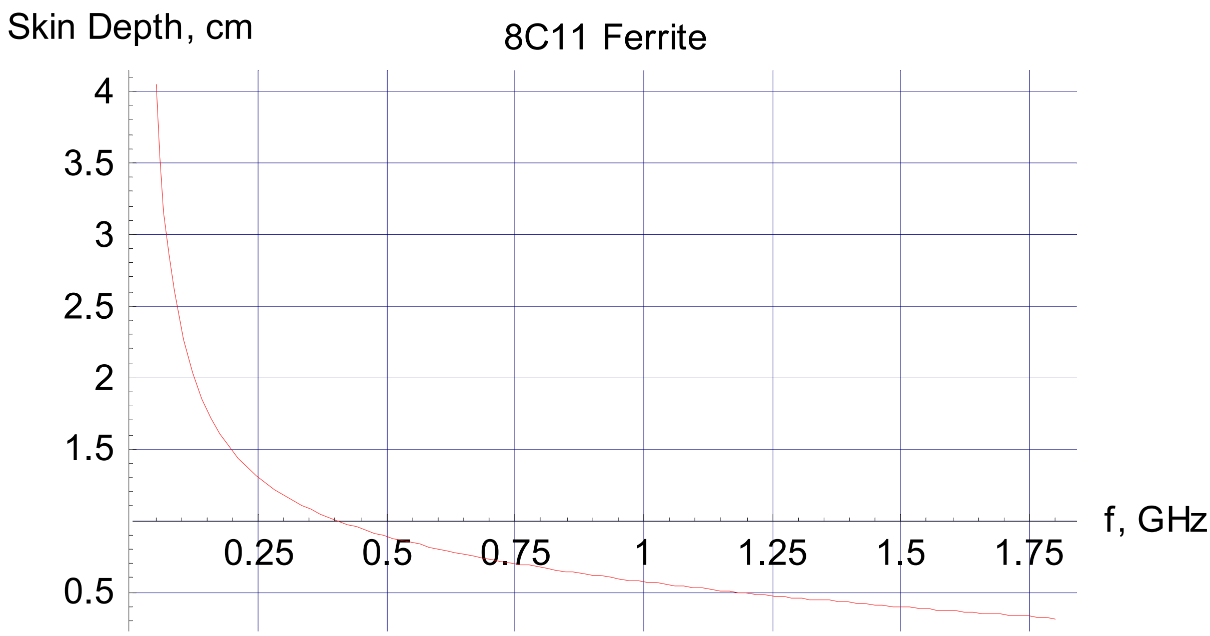

Fig. 2: Skin depth of 8C11 ferrite.

This figure shows that for the single-bunch range of frequencies $f \geq 0.1$ GHz, the skin depth is fairly small compared with the thickness ($d = 60$ mm), so the finite thickness of the kicker does not actually play a role here.

The transverse impedance of a single ferrite kicker is calculated according to Ref. [1] with the correction of Eq. (5a) above; the Yokoya factor 0.82 for the flat chamber is then applied. The impedance can be characterized as a broad-band one, centered at ~ 2 GHz with the peak value ~ 0.6 MΩ/m. The five MKE kickers installed in the SPS in 2003 should therefore contribute ~ 3 MΩ/m, which is in the range of numbers estimated from beam observations for the low-frequency inductive part [2].

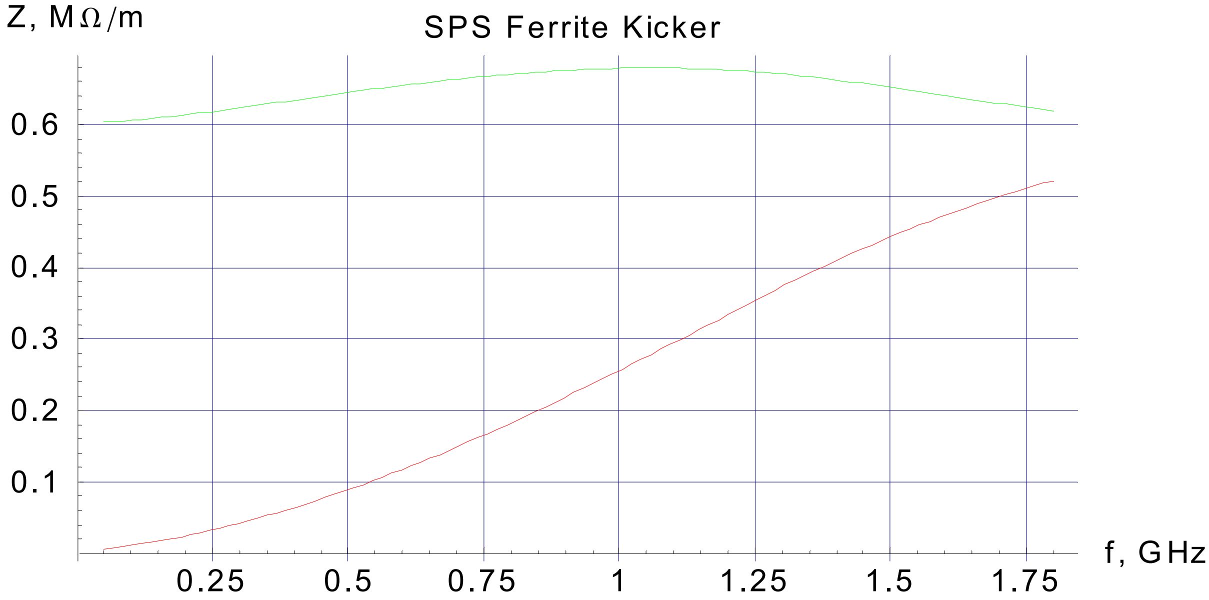

Fig. 3: Transverse impedance of the SPS MKE kicker.

3. Acknowledgements

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4. References

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